1. a. Growth constant $r = 0.09692$. The general solution is given by $P_n = 227(1.09692)^n$, where n is in decades after 1980. Populations in 2000 and 2020 are 273.1 and 328.6 million, respectively.

b. Growth constant $r = 0.2319$. Populations in 2000 and 2020 are 104.7 and 158.9 million, respectively. Mexico's population would double in 3.32 decades or 33.2 years.

c. The population of Mexico will first exceed that of U. S. in 103 years with Mexico having a population of 591.2 million and U. S. having a population of 588.6 million.

2. a. If $P_0 = 2000$, then $P_1 = -500$, $P_2 = -1437.5$, and $P_3 = -5817.4$. The equilibria are $P_e = 0$ and 1000.

b. The graph of the updating function, $f(P)$, with the identity map, $P_{n+1} = P_n$, is shown below. The P_n-intercepts are 0 and 1800. The vertex of the parabola occurs at $(900, 1012.5)$.

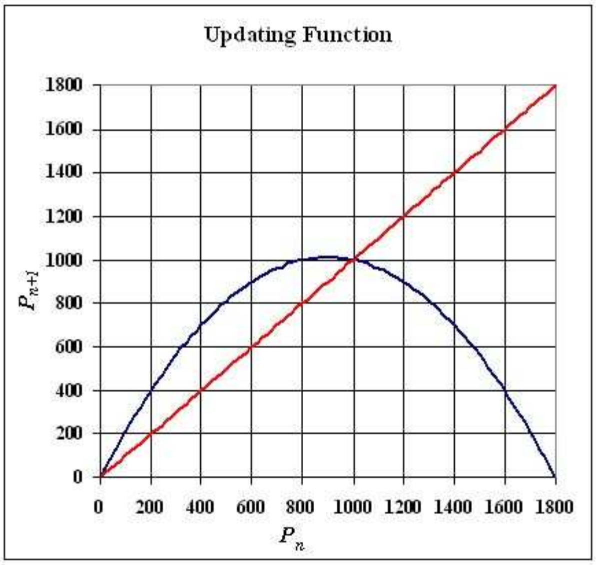

Figure 1: The identity map and the updating function intersect at the equilibria.

c. The derivative of $f(P)$ is $f'(p) = 2.25 - 0.0025P$. At the equilibrium, $P_e = 0$, $f(0) = 2.25$, which implies that this equilibrium is unstable with solutions monotonically growing away. At the equilibrium, $P_e = 1000$, $f(1000) = -0.25$, which implies that this equilibrium is stable with solutions oscillating toward the equilibrium.

3. a. With $P_0 = 500$, this discrete logistic model gives $P_1 = 516$, $P_2 = 532$, and $P_3 = 548$.

b. The updating function $f(p)$ has intercepts at

$$p = \frac{1}{2}\left(11000 \pm \sqrt{(11000)^2 - 360000}\right) \simeq 8.188, 10992.$$

The vertex occurs at $(5500, 3016)$. The equilibria are $P_e = 100$ or 900.

c. The derivative of $f(P)$ is $f'(p) = 1.1 - 0.0002P$. At the equilibrium, $P_e = 100$, $f(100) = 1.08$, which implies that this equilibrium is unstable with solutions monotonically growing away. At the equilibrium, $P_e = 900$, $f(900) = 0.92$, which implies that this equilibrium is stable with solutions monotonically approaching the equilibrium.

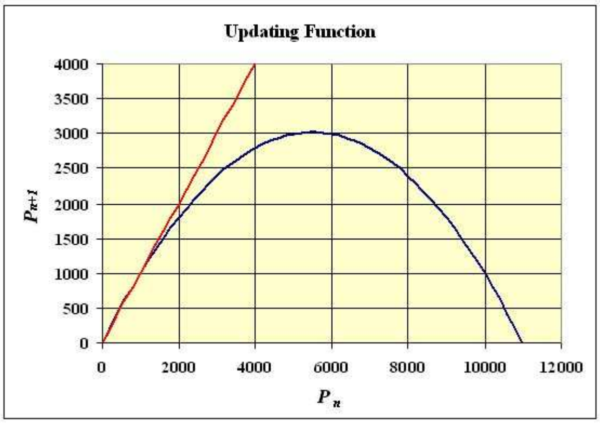

Figure 2: The identity map and the updating function intersect at the equilibria.

4. a. With $P_0 = 41.8$ M and $P_2 = 50.8$ M, the Malthusian growth model for France gives $r = 0.1024$. The general solution is given by

$$P_n = 41.8(1.1024)^n.$$

b. The model above gives the population in 2000 as $P_5 = 68.06$ M, which is 14.6% higher than the actual population of 59.4 million.

c. The logistic growth model with $P_0 = 41.8$ gives the populations in 1960 and 1970 as $P_1 = 46.235$ and $P_2 = 50.29$, respectively.

d. The equilibria for this Logistic growth model are $P_e = 0$ and 67.31. The derivative of $F(P)$ is $F'(p) = 1.28 - 0.00832P$. At the equilibrium, $P_e = 0$, $F(0) = 1.28$, which implies that this equilibrium is unstable with solutions monotonically growing away. At the equilibrium, $P_e = 67.31$, $F(67.31) = 0.72$, which implies that this equilibrium is stable with solutions monotonically approaching this equilibrium.

5. a. The value of r is $r = 0.4676$. The general solution is $P_n = 1.3(1.4676)^n$.

b. In 2000, the model predicts $P_5 = 8.85$ crabs/m^2, which gives an error of 24.7% too high an estimate.

c. The logistic growth model predicts population densities for the mitten crabs of $P_2 = 1.926$ crabs/m^2 in 1996 and $P_3 = 2.799$ crabs/m^2 in 1997.

d. The equilibria are $P_e = 0$ and 12 crabs/m^2. The derivative of the updating function is $F'(P) = 1.54 - 0.09P$. For the higher equilibrium, $F'(12) = 0.46$, so this equilibrium is stable with population densities monotonically approaching this value.

6. a. The populations are $P_1 = 800e^{-0.4} \simeq 536.26$ and $P_2 = 502.2$.

b. The derivative of $R(P)$ is $R'(P) = 8(1 - .004P)e^{-0.004P}$. The maximum of $R(P)$ occurs at $P = 250$ with $R(250) = 2000e^{-1} = 735.76$. As $P \to \infty$, the exponential dominates the polynomial part, so $R(P) \to 0$. The graph of the Ricker's function is below.

c. The equilibria are $P_e = 0$ and $P_e = 250\ln(8) = 519.86$. At $P_e = 0$, $R'(0) = 8 > 1$, so this equilibrium is unstable with solutions monotonically growing away from $P_e = 0$. At $P_e = 519.86$, $R'(519.86) = -1.079 < -1$, so this equilibrium is unstable with solutions oscillating and moving away from $P_e = 519.86$.

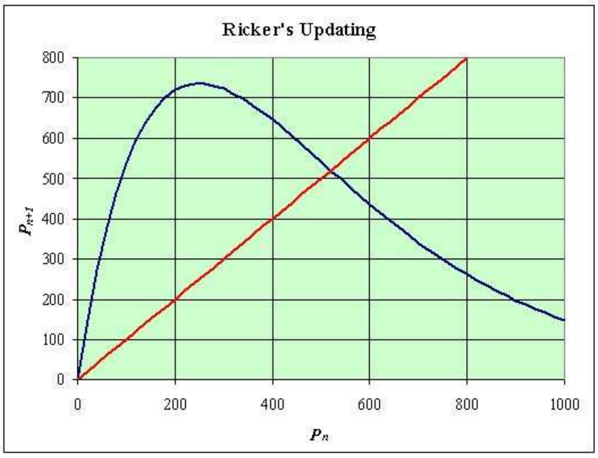

Figure 3: The identity map and the updating function intersect at the equilibria.

7. a. With $h = 0.5$, $P_1 = 402.4$ and $P_2 = 1144.3$.

b. The equilibria with $h = 0.5$ are $P_e = 0$ and $1000 \ln\left(\frac{10}{3}\right) \simeq 1204.0$. The derivative of the updating function is

$$R'(P) = 5(1 - 0.001P)e^{-0.001P} - 0.5.$$

At $P_e = 0$, $R'(0) = 4.5 > 1$, so this equilibrium is unstable, monotonically growing away from 0. At $P_e = 1204$, $R'(1204) = -0.806$, so this equilibrium is stable, oscillating toward the equilibrium.

c. Solving $P_e = 5P_e e^{-0.001P_e} - hP_e$ gives either $P_e = 0$ or

$$P_e = 1000 \ln\left(\frac{5}{1+h}\right),$$

which is zero when $h = 4$. Thus, a fishing intensity of $h \geq 4$ leads to extinction.

8. a. The next two generations are $P_1 = 800$ and $P_2 = 512$.

b. The only intercept is $(0, 0)$. There is a horizontal asymptote at $H = 0$, since $\lim_{P\to\infty} H(P) = 0$. The derivative of $H(P)$ is given by

$$H'(p) = \frac{16(1 - 0.005P)}{(1 + 0.005P)^3}.$$

The maximum occurs at $(200, 800)$. The graph is below.

c. There are two equilibria. At $P_e = 0$, $H'(0) = 16 > 1$, so this equilibrium is unstable, monotonically growing away from 0. At $P_e = 600$, $H'(600) = -0.5$, so this equilibrium is stable, oscillating toward the equilibrium.

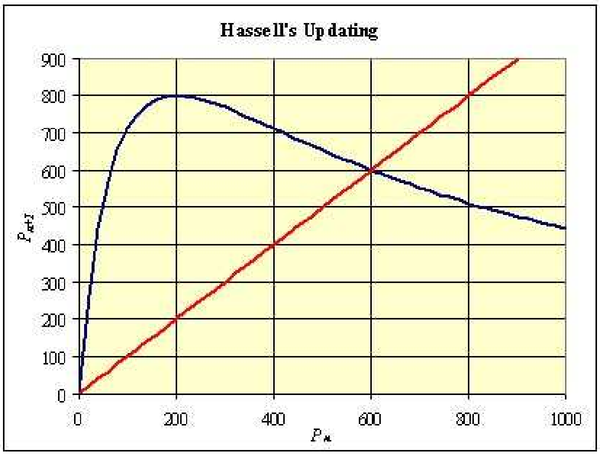

Figure 4: The identity map and the updating function intersect at the equilibria.