1. A woman with a chronic lung problem breathes a supply of air enriched with Helium. Experimental measurements show the following concentrations of the exhaled air after she resumes normal breathing in the room.

Breath Number	0	1	2
Conc. of He (ppm)	400	352	310

The concentration of Helium in the room, γ, is not known, but assumed to be constant.

a. Assume a breathing model of the form:

$$c_{n+1} = (1-q)c_n + q\gamma.$$

Use the data above to find the constants q, the fraction of air exchanged, and γ, the ambient concentration of Helium. Then determine the concentration of Helium in the next two breaths, c_3 and c_4.

b. Find the equilibrium concentration of Helium in the subject's lungs based on this breathing model. What is the stability of this equilibrium concentration?

c. Graph the updating function along with the identity map, $c_{n+1} = c_n$, showing all intercepts ($c_n \geq 0$) and points of intersection.

2. An invertebrate living in a pond is effected by a pollutant that is slowly seeping into the ecosystem. The population dynamics for this invertebrate is given by the nonautonomous Malthusian growth model

$$P_{n+1} = (1 + k(t_n))P_n \quad \text{with} \quad P_0 = 40,000,$$

where $t_n = n$ is the number of days from the initial measurement of the population and $k(t) = 0.08 - 0.01t$ is the growth rate of this invertebrate, which is clearly declining as t increases.

a. Find the population for this organism for the first 5 days.

b. When the growth rate falls to zero, this population reaches its maximum. Find when this occurs and what the population is at that time.

c. Determine when the pollution level gets so high that this invertebrate goes extinct. (Find the time when the population first drops below one individual.)

3. Many European countries are leveling off and their population will soon begin to decline as couples produce on average less than two children per couple. Italy is the slowest growing country in the world. In 1960, Italy had 50.2 million people. In 1970 and 1980, Italy had 53.7 and 56.5 million people, respectively.

a. The average growth rate for the decades listed above is 6.1% per decade. Let $P_0 = 50.2$ with $r = 0.061$ and n as the number of decades after 1960. Use the Malthusian growth model ($P_{n+1} = (1 + r)P_n$) to estimate the population of Italy in 1990 and 2000. At this growth rate, how long would it take Italy's population to double?

b. Closer examination of the data shows that the growth rate between 1960 and 1970 is 7.0%, while between 1970 and 1980 the growth rate is 5.2%. These two growth rates suggest that a declining growth rate of the form

$$k(t_n) = 3.598 - 0.0018t_n,$$

with $t_n = 1960 + 10n$. Use the Nonautonomous Malthusian Growth model

$$P_{n+1} = (1 + k(t_n))P_n,$$

with $P_0 = 50.2$ to estimate the population of Italy in 1990 and 2000. How long until this model predicts that Italy's population will level off and begin declining?

c. Census data on Italy show that its population in 1990 was 56.8 million and in 2000, it was 57.9 million. Find the percent error between the actual census data and the predictions you made in Parts a and b. Are the census data consistent with your prediction of when the Italian population will level off as computed by the Nonautonomous Malthusian Growth Model?

4. Using data from the U. S. census bureau, the table below presents the population (in millions) for France. This lab has you repeat for this country the modeling effort that we performed in class for the U. S.

Year	Population
1950	41.83
1960	45.67
1970	50.79
1980	53.87
1990	56.74
2000	59.38

a. Find the growth rate for each decade with the data above by dividing the population from one decade by the population of the previous decade and subtracting 1 from this ratio. Associate each growth rate with the earlier of the two census dates. Determine the average (mean) growth rate, r, from the data above. Associate t with the earlier of the dates in the growth ratio, and use Excel's Trendline to find the best straight line

$$k(t) = a + bt$$

through the growth data. Graph the constant function r, $k(t)$, and the data as a function of t over the period of the census data. It is very important that you click on the Trendline equation and reformat the coefficient b so that it has more significant figures (obtain 4 significant figures for a and b).

b. The Discrete Malthusian growth model is given by

$$P_{n+1} = (1 + r)P_n.$$

where r is computed in Part a. and P_0 is the population in 1950. Write the general solution to this model, where n is in decades. Use the model to predict the population in 2020 and 2050.

c. The revised growth model is given by

$$P_{n+1} = (1 + k(t_n))P_n.$$

where $k(t_n)$ is computed in Part a. and P_0 is again the population in 1950. Simulate this nonautonomous discrete dynamical model from 1950 to 2050. (Note that $t_n = 1950 + 10n$.) Use the model to predict the population in 2020 and 2050.

d. Create a table listing the date, the population data, the predicted values from the Malthusian growth model, the nonautonomous dynamical model, and the percent error between the actual population and each of the predicted populations from the models from 1950 to 2000. What is the maximum error for each model over this time interval? Use Excel to graph the data and the solutions to the each of the models above for the period from 1950 to 2050. Briefly discuss how well these models predict the population over this period. List some strengths and weaknesses of each of the models and how you might obtain a better means of predicting the population.

e. The growth rate of the nonautonomous dynamical model goes to zero during this century for France. At this time, this model predicts that the population will reach its maximum and start declining. Use the growth rate $k(t)$ to find when this model predicts a maximum population, then estimate what that maximum population will be.

f. Use the data above to find the best discrete logistic growth model fit for the population of France. Add the graph of this model to your previous graph of the Malthusian growth models for the time period 1950 to 2050. What does this model predict for the population of France in 2050? From the sum of square errors, which model matches the data best? Find all equilibria of this model and discuss the stability of these equilibria (include the values of the derivatives at the equilibria). What does this model predict will happen over a long period of time for France's population?