1. Solve the following initial value problems. Use the computer to graph the solution for $t \in [0, 10]$.

a. $\dfrac{dy}{dt} = 5 - 0.2y, \quad y(2) = 10$

b . $\dfrac{dy}{dt} = e^{-t}\left(1 + y^2\right), \quad y(0) = 0$

c. $\dfrac{dy}{dt} = t^3 - 2ty, \quad y(0) = 10$

d. $\dfrac{dy}{dt} = \dfrac{y^2 - 2ty}{t^2}, \quad y(1) = 2$

2. Consider the following initial value problem:

$$\frac{dy}{dt} = y - t^2, \qquad y(0) = 1.$$

a. Solve this differential equation.

b. Show the slope field for $t \in [-5, 5]$ and $y \in [-5, 5]$, including the solution of the initial value problem.

c. Use Eulers method to simulate the solution for $t \in [0, 5]$ with a stepsize of $h = 0.2$ and $h = 0.1$. Create a table comparing the actual solution to each of these numerical simulations at times $t = 1, 2, 3, 4$, and 5. (Dont write all your calculations needed for Eulers method!) Graph the actual solution and each of the Eulers simulations.

3. Consider the following initial value problem:

$$\frac{dy}{dt} = y^2 + t^2, \qquad y(0) = 1.$$

a. Show the slope field for $t \in [-2, 2]$ and $y \in [-5, 5]$, including the solution of the initial value problem.Solve this differential equation.

b. Use Maple to find the actual solution. Determine when the solution becomes unbounded. (You might want to solve for $1/y(t) = 0$ to find this vertical asymptote.)

c. Use Eulers method to simulate the solution for $t \in [0, 1]$ with a stepsize of $h = 0.05$ and $h = 0.01$. Create a table comparing the actual solution to each of these numerical simulations at times $t = 0.5, 0.8$, and 0.9. (Dont write all your calculations needed for Eulers method!) Graph the actual solution and each of the Eulers simulations for $t \in [0, 1]$.

4. For each of the following problems, locate the bifurcation values for the one-parameter family and draw phase lines for values of the parameter slightly smaller than, slightly larger than, and at the bifurcation value.

a. $\dfrac{dy}{dt} = \alpha y - y^2$

b . $\dfrac{dy}{dt} = -y + \tanh(\alpha y)$

5. a. An initially clean lake ($c(0) = 0$) maintains a constant volume of $V = 10^5$ m^3 of water. There are two streams entering this lake with differing concentrations of agricultural pesticide. The first stream has a flow rate of $f_1 = 200$ m^3/day with a pesticide concentration of $Q_1 = 7\,\mu$g/m^3. A second stream has a flow rate of $f_2 = 300$ m^3/day with a pesticide concentration of $Q_2 = 2\,\mu$g/m^3. Assume that this is a well-mixed lake with a stream flowing out at a rate of $f_3 = 500$ m^3/day (with the pesticide in that stream equal to the concentration in the lake). Write a differential equation describing the concentration of pesticide in the lake ($c(t)$) and solve this differential equation.

b. Determine how long until the lake has a concentration of $2\,\mu$g/m^3 of pesticide. Also, find the limiting concentration of pesticide in this lake. Graph the solution for one year.

6. a. The U. S. population was 76.0 million in 1900 and 105.7 million in 1920. Use the Malthusian growth model $P' = rP$ to represent the population of the U. S. Solve this differential equation, assuming that $t = 0$ is 1900 and with the data above find the value of r to 4 significant figures. How long does it take for the population to double with this model?

b. The population of U. S. was 92.0 million in 1910 and 151.3 million in 1950. Find the percent error between the Malthusian growth model predictions and these data.

c. A logistic growth model for the U. S. population that reasonably fits the census data for the 20^{th} century is given by

$$\frac{dP}{dt} = 0.02P\left(1 - \frac{P}{420}\right), \qquad P(0) = 76.0.$$

where t is in years after 1900. Solve this differential equation, then find the percent error between the actual population and this model for the years 1910, 1920, and 1950. How long does it take for the 1900 population to double with this model?

d. Determine the equilibria for this logistic growth model. At what population does this model predict the U. S. will level off?