1. Solve the following initial value problems.

a. $\frac{dy}{dt} = \sin(t)y + 4\sin(t), \quad y(0) = 5$

b . $\frac{dy}{dt} = \frac{2y^2}{t+2}, \quad y(0) = 4$

c. $\frac{dy}{dt} = -0.1y + 6, \quad y(0) = 20$

d. $t\frac{dy}{dt} = 2y + t^3\cos(4t), \quad y(\pi) = 2$

e. $\frac{dy}{dt} = -\frac{y}{3}, \quad y(2) = 15$

f. $\frac{dy}{dt} = e^{t-y}, \quad y(0) = 3$

2. Consider the following initial value problem:

$$\frac{dy}{dt} = t - y, \qquad y(0) = 3.$$

a. Solve this differential equation.

b. Use Euler's method to simulate the solution for $t \in [0, 2]$ with a stepsize of $h = 0.5$.

c. Find the percent error between the Euler approximation and the actual solution at $t = 2$.

3. Consider the following initial value problem:

$$\frac{dy}{dt} = y + e^t, \qquad y(0) = -1.$$

a. Solve this differential equation.

b. Use Euler's method to simulate the solution for $t \in [0, 2]$ with a stepsize of $h = 0.5$.

c. Find the percent error between the Euler approximation and the actual solution at $t = 2$.

4. Sketch the phase lines for the given differential equations. Identify the equilibrium points as sinks, sources, or nodes.

a. $\frac{dy}{dt} = (y+2)(y-2)^3$

b . $\frac{dy}{dt} = 16y^2 - y^4$

c. $\frac{dy}{dt} = y\sin(y)$

5. For each of the following problems, locate the bifurcation values for the one-parameter family and draw phase lines for values of the parameter slightly smaller than, slightly larger than, and at the bifurcation value.

a. $\dfrac{dy}{dt} = \alpha y - 4y^3$ b . $\dfrac{dy}{dt} = \alpha - \cosh(y)$

6. You are attending a conference, and the talks are going past the coffee break time. You really need a cup of tea (not liking coffee) to keep awake for the next set of talks. The refreshments are in a room that has a constant temperature of 21°C, and you find that the hot water is only 85°C. Five minutes later, the hot water is only 81°C.

a. Assume that the container of water satisfies Newton's law of cooling. ($H' = -k(H - T_e)$, where T_e is the environmental temperature.) If it was placed out when the talks were supposed to end with boiling water (water at 100°C), then how many minutes beyond the scheduled time did the talks go? (Hint: If $H(t)$ is the temperature, then use $H(0) = 85$ and $H(5) = 81$ to find the cooling constant k in Newton's law of cooling, then find when $H(t) = 100$.)

b. If tea needs water that is at least 93°C to give you enough caffeine for the next set of talks, then how long after the scheduled end of the talks can you wait?

7. If $i(t)$ is the fraction of infectious people in a community with an air-borne disease that imparts no immunity, then the fraction of susceptible people is $1 - i(t)$, where t is in days. Assume that new infectious people are added at a rate $\alpha i(1 - i)$ with $\alpha = 0.1$ and infectious people are cured at a rate βi with $\beta = 0.07$. The model for the fraction of infectious people satisfies the differential equation:

$$\frac{di}{dt} = \alpha i(1 - i) - \beta i.$$

a. Find all equilibria for this model and draw the phase-line diagram. Identify the equilibria as sinks, sources, or nodes, then discuss what happens to the disease.

b. Assume that initially 10% of the population is infected or $i(0) = 0.1$. Solve this differential equation.

c. New drugs are being researched that improve the cure rate of this disease. Sketch the bifurcation diagram for this differential equation with $\alpha = 0.1$ and $\beta \geq 0$ varying as the bifurcation parameter. Find the cure rate β that is required to drive this disease to extinction.