Find the derivative for each of the following functions.

1. \( f(x) = x^4 + 7x^3 - 2x^2 - 4x + 3 \)
2. \( g(x) = 3x^2 - 3x + 4 - 2x^{-3} \)

3. \( h(t) = t^3 - 5t + \frac{1}{2} - \frac{1}{t^2} \)
4. \( k(z) = \frac{3z^2}{2} + 6z - \sqrt{z} \)

5. \( p(z) = z^{\frac{1}{3}} + 4.7z^2 - 7\sqrt{z^5} \)
6. \( q(w) = 3w^{-0.4} + 2.1w^5 - \frac{2}{\sqrt{w}} \)

7. \( f(x) = ax^2 + bx + c \)
8. \( g(x) = A - \frac{B}{x^3} + \frac{C}{\sqrt{x}} - Dx^4 \)

9. In the linear section, we found that the growth of a child satisfies the equation

\[ h(a) = 6.46a + 72.3 \]

where the age, \( a \), is in years and the height, \( h \), is in cm.

a. Find \( dh/da \). What is the growth rate at age 2? At age 6?

b. If a child is 135 cm at age 10, what is the predicted height at age 11?
10. The lecture notes showed that the number of species of herpatofauna, \( N \) on Caribbean Islands as a function of the area in square miles, \( A \), is approximated by the formula

\[
N = 3A^{\frac{1}{3}}.
\]

a. Find the rate of change in number of species as a function of area, \( dN/dA \), when the area of the island is 64, 125, and 1000 square miles.

b. Sketch a graph of the derivative, \( dN/dA \), for \( 0 \leq A \leq 1000 \).

11. A ball falling under the influence of gravity without air resistance satisfies the equation

\[
y(t) = -4.9t^2,
\]

where \( y \) is in meters and \( t \) is in seconds.

a. Find an expression for the velocity, \( v(t) = y'(t) \).

b. What is the velocity at \( t = 1 \) and \( t = 5 \)?

12. A ball that is thrown vertically falling under the influence of gravity without air resistance from a 128 ft platform with an upward velocity of 32 ft/sec satisfies the equation

\[
h(t) = 128 + 32t - 16t^2,
\]

where \( h \) is in feet and \( t \) is in seconds.

a. Find an expression for the velocity, \( v(t) = h'(t) \). Determine when the velocity is zero, then determine the maximum height of the ball. What is the velocity at \( t = 2 \) and \( t = 4 \).

b. Sketch a graph of \( h(t) \), showing crucial points, including the \( h \)-intercept, the maximum height, and when the ball hits the ground.
13. Suppose that a population of insects, $P$ (individuals), in a controlled experiment with constant food supply is shown to have a growth rate that fits the logistic growth function

$$g(P) = 0.04P(1 - P/800),$$

where the time is measured in days.

a. Find the population when the growth rate $g(P)$ is zero (the $P$-intercepts). This gives the equilibria for this experiment. Give a biological interpretation for each of the equilibria.

b. Compute the derivative of $g(P)$, then determine when $g(P)$ has a maximum growth rate. (State the units for the growth rate.) Give a biological interpretation of this maximum growth rate. Sketch the graph of $g(P)$.

14. An experiment with a sample of paramecium, $P$ (individuals/ml), with a fixed food supply is shown to satisfy the logistic growth function

$$G(P) = 0.02P(1 - 0.025P),$$

where the time is in hours.

a. Find the equilibria for this population, i.e., when the growth rate $G(P)$ is zero. Give a biological interpretation for each of the equilibria.

b. Compute the derivative of $G(P)$, then determine when $G(P)$ has a maximum growth rate. (State the units for this growth rate.) Give a biological interpretation of this maximum growth rate. Sketch the graph of $G(P)$. 
15. A cat is crouching on a ledge that is 12 feet above the ground, trying to ambush pigeons that fly by.

a. Suppose that a pigeon flies by 4 feet above the cat, and that the cat jumps off the ledge with just enough vertical velocity, $v_0$ to catch the pigeon. If the height of the cat is given by

$$h(t) = -16t^2 + v_0t + 12,$$

then find the velocity $v(t) = h'(t)$ of the cat at any time, $t \geq 0$.

b. Find when the velocity is equal to zero in terms of $v_0$. This is the time at the maximum height.

c. Since the cat is 16 ft in the air at this time, use the equation for the height of the cat, $h(t)$ to compute the initial velocity of the cat, $v_0$. Substitute this into the velocity equation, $v(t)$ to give the velocity of the cat at any time between jumping and hitting the ground. What is the velocity of the cat after 1 second?

d. Find when the cat hits the ground with the pigeon and what is the velocity of the cat that it hits the ground.

16. a. Lizards are cold-blooded animals whose temperatures roughly match the surrounding environment. Suppose the body temperature, $T(t)$, of a lizard is measured for a period of 18 hours from midnight until 6 PM. The body temperature (in °C) of the lizard over this period of time (in hours) is found to be well approximated by the polynomial

$$T(t) = -0.01t^3 + 0.285t^2 - 1.80t + 15.$$

Find the general expression for the rate of change of body temperature per hour ($\frac{dT}{dt}$).

b. Use this information to find what the rate of change of body temperature is at midnight, 4 AM, 8 AM, noon, and 4 PM. Which of these times gives the fastest increase in the body temperature and which shows the most rapid cooling of the lizard?