1. a. If \( e^a = 3.7 \) and \( e^b = 0.4 \), then
\[
\frac{(e^0 + e^a)^2}{e^{a-b}} = \frac{e^b(1 + e^a)^2}{e^a} = \frac{0.4(1 + 3.7)^2}{3.7} = \frac{0.4(4.7)^2}{3.7} = 2.3881.
\]

b. If \( \ln(c) = -1.5 \) and \( \ln(d) = 2 \), then
\[
\frac{\ln(d^2/c) - \ln(e)}{\ln(cd) + \ln(1)} = \frac{\ln(d^2/c) - \ln(c) - 1}{\ln(c) + \ln(d) - 0} = \frac{2\ln(d) - \ln(c) - 1}{\ln(c) + \ln(d)} = \frac{2(2.1) - (-1.5) - 1}{-1.5 + 2.1} = \frac{4.7}{0.6} = 7.833.
\]

4. The \( x \)-intercept would satisfy \( f(x) = 2 + e^{2x} = 0 \), but both 2 and \( e^{2x} \) are positive, \( f(x) = 0 \) is impossible and no \( x \)-intercept exists. The \( y \)-intercept is where \( x = 0 \), so \( f(0) = 2 + e^0 = 3 \). Thus, it occurs at \((0, 3)\). The exponential is defined for all \( x \), so there are no vertical asymptotes. A horizontal asymptote is found by examining \( x \to -\infty \). As \( x \to -\infty \), \( e^{2x} \to 0 \), so \( y \to 2 + 0 = 2 \). It follows that there is a horizontal asymptote (to the left) at \( y = 2 \). The graph is shown below to the left.
5. The $x$-intercept satisfies $f(x) = 10 - e^{-x/2} = 0$, so $e^{-x/2} = 10$. By taking the logarithms of both sides, it follows that $-\frac{x}{2} = \ln(10)$, so $x = -2\ln(10) \approx -4.605$ and the $x$-intercept is $(-2\ln(10), 0)$. The $y$-intercept is where $x = 0$, so $f(0) = 10 - e^0 = 9$. Thus, it occurs at $(0,9)$. The exponential is defined for all $x$, so there are no vertical asymptotes. A horizontal asymptote is found by examining $x \to +\infty$. As $x \to +\infty$, $e^{-x/2} \to 0$, so $y \to 10 - 0 = 10$. It follows that there is a horizontal asymptote (to the right) at $y = 10$. The graph is shown below to the left.

![Graph 1](image1.png)

7. The domain requires that the argument of the logarithm is positive, so $2x > 0$ or $x > 0$. The $x$-intercept satisfies $f(x) = \ln(2x) = 0$, so $\ln(2x) = 0$ or $2x = e^0 = 1$ Thus, $x = \frac{1}{2}$, and the $x$-intercept occurs at $(\frac{1}{2}, 0)$ The $y$-intercept is where $x = 0$, but this is outside the domain, hence doesn’t exist. A vertical asymptote occurs on the edge of the domain or at $x = 0$. The graph is shown above to the right.

![Graph 2](image2.png)
10. The domain requires that the argument of the logarithm is positive, so $1 - x > 0$ or $x < 1$. The $x$-intercept satisfies $f(x) = 2 - \ln(1 - x) = 0$, so $\ln(1 - x) = 2$ or $1 - x = e^2$. Thus, $x = 1 - e^2$, and the $x$-intercept occurs at $(1 - e^2, 0)$. The $y$-intercept is where $x = 0$, so $f(0) = 2 - \ln(1 - 0) = 2$ or $(0, 1)$. A vertical asymptote occurs on the edge of the domain or at $x = 1$. The graph is shown below.

11. a. The average number of mammalian species satisfies $N = kA^{\frac{1}{3}}$, so if the islands have areas of 125 and 8000 km$^2$, then

$$N(125) = 2(125)^{\frac{1}{3}} = 2(5) = 10,$$

$$N(8000) = 2(8000)^{\frac{1}{3}} = 2(20) = 40.$$  

b. For 32 species of mammals, the island area satisfies $N = 32 = 2A^{\frac{1}{3}}$, so $16 = A^{\frac{1}{3}}$ and $A = 16^3 = 4096$ km$^2$.

c. The graph is below.
14. a. The allometric model satisfies:

\[ P = kW^a, \]
\[ \ln(P) = a \ln(W) + \ln(k). \]

Note that if \( Y = \ln(P) \), \( X = \ln(W) \), and \( K = \ln(k) \), then this is just the equation of a line

\[ Y = aX + K, \]

where \( X \) and \( Y \) are the logarithms of the data.

The data given is shown with their logarithmic values in the table below:

<table>
<thead>
<tr>
<th>W</th>
<th>ln(W)</th>
<th>P</th>
<th>ln(P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.386</td>
<td>615</td>
<td>6.422</td>
</tr>
<tr>
<td>28</td>
<td>3.332</td>
<td>350</td>
<td>5.858</td>
</tr>
</tbody>
</table>

From the formula above, the slope is \( a \) and satisfies

\[ a = \frac{\ln(P_2) - \ln(P_1)}{\ln(W_2) - \ln(W_1)} = \frac{5.858 - 6.422}{3.332 - 1.386} = -0.2897. \]

To obtain \( k \), we see that

\[ \ln(k) = \ln(P_1) - a \ln(W_1) = 6.422 + 0.2897(1.386) = 6.823, \]

so

\[ k = e^{\ln(k)} = e^{6.823} = 918.9. \]

This gives the allometric model

\[ P = 918.9W^{-0.2897}. \]

b. For an 11 gram wren, the allometric model gives:

\[ P = 918.9(11)^{-0.2897} = 459 \text{ beats/min}. \]

If a dove has a pulse of 130 beats/min, then the allometric model gives

\[ 130 = 918.9W^{-0.2897} \]

\[ W^{0.2897} = \frac{918.9}{130} \]

\[ W = \left( \frac{918.9}{130} \right)^{1/0.2897} = 855 \text{ g} \]
15. a. The allometric model satisfies:

\[ T = kw^a, \]
\[ \ln(T) = a \ln(w) + \ln(k). \]

Note that if \( Y = \ln(T), \ X = \ln(w), \) and \( K = \ln(k), \) then this is just the equation of a line

\[ Y = aX + K, \]

where \( X \) and \( Y \) are the logarithms of the data.

The data given is shown with their logarithmic values in the table below:

<table>
<thead>
<tr>
<th>( w )</th>
<th>( \ln(w) )</th>
<th>( T )</th>
<th>( \ln(T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>4.248</td>
<td>120</td>
<td>4.787</td>
</tr>
<tr>
<td>1.5</td>
<td>0.4055</td>
<td>65</td>
<td>4.174</td>
</tr>
</tbody>
</table>

From the formula above, the slope is \( a \) and satisfies

\[ a = \frac{\ln(T_2) - \ln(T_1)}{\ln(w_2) - \ln(w_1)} = \frac{4.787 - 4.174}{4.248 - 0.4055} = 0.1595. \]

To obtain \( k, \) we see that

\[ \ln(k) = \ln(T_1) - a \ln(w_1) = 4.787 - 0.1595(4.248) = 4.110, \]

so

\[ k = e^{\ln(k)} = e^{4.110} = 60.93. \]

This gives the allometric model

\[ T = 60.93w^{0.1595}. \]

b. For a 20 kg dog, the allometric model gives:

\[ T = 60.93(20)^{0.1595} = 98.3 \text{ days}. \]

If an animal has erythrocytes with a lifetime of 100 days, then the allometric model gives

\[
\frac{100}{w^{0.1595}} = \frac{100}{60.93} = 22.3 \text{ kg}
\]