This Lecture Activity has you actively work with the lecture notes presented in class and available on my website. This activity is due by **Mon. Oct 4 by noon**. The problems below require written answers, which are entered into **Gradescope**.

Note: For full credit you must show intermediate steps in your calculations.

All of the matrices below are *companion matrices*. It follows that an eigenvector for an eigenvalue, λ, is $\mathbf{v} = [1, \lambda, \lambda^2, ..., \lambda^n]^T$. In all of the problems below use this information to obtain the columns of the transformation matrix, P. For complex eigenvalues the real and imaginary parts of $\mathbf{v}$ form two of the columns. This vector is also used to obtain vectors in the higher null spaces.

1. (8pts) Consider the companion matrix:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -9 & -18 & -10 & -2 \end{pmatrix}.$$

Write the characteristic equation and give the eigenvalues and corresponding eigenvectors, noting their algebraic and geometric multiplicities. Using the form noted above ($\mathbf{v}$), find the *linear transformation matrix*, P, such that

$$P^{-1}AP = J,$$

where J is a real matrix in real Jordan form. Show how you obtain any eigenvectors in a higher null space if they are required. Give the matrix J. (You don't have to show P^{-1}.)

Now consider the system of linear ODEs

$$\dot{\mathbf{y}} = J\mathbf{y}.$$

Give the real fundamental solution to this ODE, $\Psi(t)$. (Slide Fundamental 27–46)

2. (8pts) Consider the companion matrix:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 25 & 15 & -14 & -18 & -7 \end{pmatrix}.$$

Repeat the steps in Problem 1 to find eigenvalues and eigenvectors with their multiplicities, P, J, and the real fundamental solution, $\Psi(t)$.