This assignment is your second Lecture Activity to have you actively work with the lecture notes presented in class and available on my website. This activity is due by **Mon. Sep 6 by noon**. There are 4 problems that require written answers, which are entered into **Gradescope**.

Note: For full credit you must show intermediate steps in your calculations.

1. (3pts) We examined carbon radiodating, using the isotope ^{14}C, which has a half-life of 5730 yr. Find the age of a fossilized bone from a man in Western Pennsylvania that contains 16% of its original ^{14}C. When did a Kenyan man die if his bone contains 8% of its original ^{14}C? Suppose the error in each of the readings above is $\pm 1\%$. Find the range of possible ages for each of the bones. (Slides Linear 3-5)

2. (6pts) Consider a hypothetical cascade of radioactive elements, x_0, x_1, x_2, and x_3 with x_4 a stable element.

$$x_0 \xrightarrow{h_0} x_1 \xrightarrow{h_1} x_2 \xrightarrow{h_2} x_3 \xrightarrow{h_3} x_4$$

The half-lives are given by h_i with

$$h_1 = 1 \text{ da}, \qquad h_2 = 10 \text{ da}, \qquad h_3 = 400 \text{ da}.$$

Define the decay rates $k_i = \frac{\ln(2)}{h_i}$, $\quad i = 1, 2, 3$, then the linear nonhomogeneous system of ODEs satisfies:

$$\dot{x}_i = k_{i-1}x_{i-1} - k_i x_i, \qquad i = 1, 2, 3,$$

where $r = k_0 x_0$ is the rate of decay of x_0, which is assumed to be constant. Let $r = 10$. With $\mathbf{x} = [x_1, x_2, x_3]^T$, this can be written as the ODE system:

$$\dot{\mathbf{x}} = A\mathbf{x} + B,$$

where

$$A = \begin{pmatrix} -k_1 & 0 & 0 \\ k_1 & -k_2 & 0 \\ 0 & k_2 & -k_3 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

Solve this cascading system of ODEs with the initial condition:

$$[x_1(0), x_2(0), x_3(0)]^T = [0, 0, 0]^T.$$

Find the values of $[x_1(t), x_2(t), x_3(t)]^T$ at $t = 100$ and $t = 400$. (Slides Linear 7-8)

3. (4pts) Continuing Problem 2: We see that the half-lives h_1 and h_2 are significantly shorter than h_3, so the time scales suggest that one can implement a *quasi-steady state* approximation for $\dot{x}_1$ and $\dot{x}_2$. Use this approximation to reduce the system of equations into a scalar ODE in $\dot{x}_3$. Solve this scalar equation in x_3. Find $x_3(t)$ at $t = 100$ and $t = 400$ and compare your answers to Problem 2. (The algebraic equations will provide the values for x_1 and x_2.) (Slide 9)

4. (3pts) Solve the linear ODE given by:

$$t\frac{dy}{dt} - y = 3t^2\sin(t).$$

(Slides Linear 17-21)