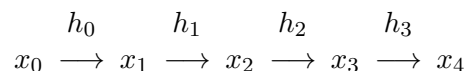


This assignment is your second Lecture Activity to have you actively work with the lecture notes presented in class and available on my website. This activity is due by **Mon. Sep 6 by noon**. There are 4 problems that require written answers, which are entered into **Gradescope**.

**Note:** For full credit you must show intermediate steps in your calculations.

1. (3pts) We examined carbon radiodating, using the isotope  $^{14}\text{C}$ , which has a half-life of 5730 yr. Find the age of a fossilized bone from a man in Western Pennsylvania that contains 16% of its original  $^{14}\text{C}$ . When did a Kenyan man die if his bone contains 8% of its original  $^{14}\text{C}$ ? Suppose the error in each of the readings above is  $\pm 1\%$ . Find the range of possible ages for each of the bones. (Slides Linear 3-5)

2. (6pts) Consider a hypothetical cascade of radioactive elements,  $x_0$ ,  $x_1$ ,  $x_2$ , and  $x_3$  with  $x_4$  a stable element.



The half-lives are given by  $h_i$  with

$$h_1 = 1 \text{ da}, \quad h_2 = 10 \text{ da}, \quad h_3 = 400 \text{ da}.$$

Define the decay rates  $k_i = \frac{\ln(2)}{h_i}$ ,  $i = 1, 2, 3$ , then the linear nonhomogeneous system of ODEs satisfies:

$$\dot{x}_i = k_{i-1}x_{i-1} - k_i x_i, \quad i = 1, 2, 3,$$

where  $r = k_0 x_0$  is the rate of decay of  $x_0$ , which is assumed to be constant. Let  $r = 10$ . With  $\mathbf{x} = [x_1, x_2, x_3]^T$ , this can be written as the ODE system:

$$\dot{\mathbf{x}} = A\mathbf{x} + B,$$

where

$$A = \begin{pmatrix} -k_1 & 0 & 0 \\ k_1 & -k_2 & 0 \\ 0 & k_2 & -k_3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

Solve this cascading system of ODEs with the initial condition:

$$[x_1(0), x_2(0), x_3(0)]^T = [0, 0, 0]^T.$$

Find the values of  $[x_1(t), x_2(t), x_3(t)]^T$  at  $t = 100$  and  $t = 400$ . (Slides Linear 7-8)

3. (4pts) Continuing Problem 2: We see that the half-lives  $h_1$  and  $h_2$  are significantly shorter than  $h_3$ , so the time scales suggest that one can implement a *quasi-steady state* approximation for  $\dot{x}_1$  and  $\dot{x}_2$ . Use this approximation to reduce the system of equations into a scalar ODE in  $\dot{x}_3$ . Solve this scalar equation in  $x_3$ . Find  $x_3(t)$  at  $t = 100$  and  $t = 400$  and compare your answers to Problem 2. (The algebraic equations will provide the values for  $x_1$  and  $x_2$ .) (Slide 9)

4. (3pts) Solve the linear ODE given by:

$$t \frac{dy}{dt} - y = 3t^2 \sin(t).$$

(Slides Linear 17-21)