Math 636

Begin by working the HW problems assigned in WeBWorK. Some of those questions ask for graphs and written explanations (usually noted "In your written HW..."), so you will need to include that information in your HW assignment turned in with the problem below. All answers need to be written in a clear, succinct manner. Write a brief paragraph summarizing the answers to the problems with each answer clearly stated in a sentence. Supporting graphs should be provided when asked for, but you should not include printouts of spreadsheets. You can create an appendix to a problem, but that should only include significant material to back up your answers.

Beverton-Holt Discrete Population Model:

a. The discrete Malthusian growth model given by,

$$P_{n+1} = (1+r)P_n,$$

with initial population, P_0 , has the explicit solution,

$$P_n = P_0 (1+r)^n.$$

Consider the discrete Malthusian growth model with immigration given by,

$$P_{n+1} = (1+r)P_n + \mu,$$

with initial population, P_0 . Give the solutions P_2 and P_4 in terms of the parameters, r, μ , and P_0 . Extend this result to give the solution, P_n in terms of r, μ , n, and P_0 . Assume r > 0 and find all equilibria for this model. Discuss the stability of the equilibria. Be sure to include the modeling significance of your results.

b. The Beverton-Holt population model is given by the discrete model

$$P_{n+1} = \frac{aP_n}{1+bP_n} \equiv B(P_n),$$

where the initial population, P_0 is specified. Use a Maclaurin series expansion for $B(P_n)$ to relate the parameter a to the parameter r in the discrete Malthusian growth model for small P_0 . Assume r > 0 and find all equilibria for this model. Discuss the stability of the equilibria. Be sure to include the modeling significance of your results.

c. Find an explicit solution to the Beverton-Holt model. That is, determine P_n in terms of the parameters, a, b, n, and P_0 . **Hint:** You may want to begin by examining the Beverton-Holt model with a reciprocal transformation of the form

$$P_n = \frac{1}{Y_n}.$$