I-Clicker Question

Which of the following optimization Homework or Review problems would you most like to see worked in Lecture?

A. **Homework 3 or 4**: Optimal dimensions of an open box.

B. **Homework 11 or Review 24**: Optimal time of escape (otter or rabbit).

C. **Homework 12**: Optimal lamp illumination.

D. **Review 23**: Optimal size of brochure.

E. **Review 26**: Optimal size of holding pens.

Joseph M. Mahaffy, ⟨jmahaffy@mail.sdsu.edu⟩
Optimization – Homework — (1/12)
Optimization problem seeks to find the maximum illumination by changing $h$ if

$$I = \frac{3.3 \cos(\theta)}{d^2}$$
Given the diagram for the table, find \( d \).

A. \( d = \frac{4}{\tan(\theta)} \)

B. \( d = 4 \tan(\theta) \)

C. \( d = 4 \sin(\theta) \)

D. \( d = \frac{4}{\sin(\theta)} \)

E. \( d = \frac{4}{\cos(\theta)} \)
Given that

\[ I = \frac{3.3 \cos(\theta)}{d^2} \quad \text{and} \quad d = \frac{4}{\sin(\theta)} \]

It follows that

\[ I = \frac{3.3}{16} \cos(\theta) \sin^2(\theta) \]
Given

\[ I(\theta) = \frac{3.3}{16} \cos(\theta) \sin^2(\theta) \]

**Find the derivative of** \( I(\theta) \)?

A. \( I'(\theta) = \frac{3.3}{16} \sin(\theta)(\cos^2(\theta) - \sin^2(\theta)) \)

B. \( I'(\theta) = -\frac{6.6}{16} \sin^2(\theta) \cos(\theta) \)

C. \( I'(\theta) = \frac{3.3}{16} \sin(\theta)(3 \cos^2(\theta) - 1) \)

D. \( I'(\theta) = \frac{3.3}{16} \sin(\theta)(\sin^2(\theta) - 2 \cos^2(\theta)) \)

E. \( I'(\theta) = \frac{3.3}{16} \cos(\theta)(2 \cos^2(\theta) - \sin^2(\theta)) \)

**Hint:** You may need to use the identity \( \cos^2(\theta) + \sin^2(\theta) = 1 \)
Lamp Problem

For

\[ I = \frac{3.3 \cos(\theta)}{d^2} \]

We have

\[ I'(\theta) = \frac{3.3}{16} \sin(\theta) (3 \cos^2(\theta) - 1) \]
Lamp Problem

For

\[ I = \frac{3.3 \cos(\theta)}{d^2} \]

We have

\[ I'(\theta) = \frac{3.3}{16} \sin(\theta)(3 \cos^2(\theta) - 1) \]

The optimal solution satisfies

\[ \frac{3.3}{16} \sin(\theta)(3 \cos^2(\theta) - 1) = 0 \]
Lamp Problem

For

\[ I = \frac{3.3 \cos(\theta)}{d^2} \]

We have

\[ I'(\theta) = \frac{3.3}{16} \sin(\theta)(3 \cos^2(\theta) - 1) \]

The optimal solution satisfies

\[ \frac{3.3}{16} \sin(\theta)(3 \cos^2(\theta) - 1) = 0 \]

It follows that either

\[ \sin(\theta) = 0 \quad \text{or} \quad 3 \cos^2(\theta) - 1 \]
Lamp Problem

For

\[ I = \frac{3.3 \cos(\theta)}{d^2} \]

We have

\[ I'(\theta) = \frac{3.3}{16} \sin(\theta)(3 \cos^2(\theta) - 1) \]

The optimal solution satisfies

\[ \frac{3.3}{16} \sin(\theta)(3 \cos^2(\theta) - 1) = 0 \]

It follows that either

\[ \sin(\theta) = 0 \quad \text{or} \quad 3 \cos^2(\theta) - 1 \]

Since \( \sin(\theta) = 0 \) implies \( \theta = 0 \), which is not optimal, it follows that

\[ \cos(\theta) = \frac{1}{\sqrt{3}} \]
Given

\[
\cos(\theta) = \frac{1}{\sqrt{3}} = \frac{h}{d}
\]

we need \(\sin(\theta)\). **What is** \(\sin(\theta)\)?

A. \(\sin(\theta) = \frac{2}{\sqrt{3}}\)

B. \(\sin(\theta) = \frac{2}{3}\)

C. \(\sin(\theta) = \frac{\sqrt{3}}{2}\)

D. \(\sin(\theta) = \frac{1}{2}\)

E. \(\sin(\theta) = \sqrt{\frac{2}{3}}\)
We combine our results.

- The optimal solution has

\[ \cos(\theta_{opt}) = \frac{1}{\sqrt{3}} = \frac{h}{d} \]
Lamp Problem

We combine our results.

- The optimal solution has

\[
\cos(\theta_{opt}) = \frac{1}{\sqrt{3}} = \frac{h}{d}
\]

- The hypotenuse is

\[
d = \frac{4}{\sin(\theta_{opt})} \quad \text{with} \quad \sin(\theta_{opt}) = \sqrt{\frac{2}{3}}
\]
We combine our results.

- The optimal solution has

\[
\cos(\theta_{opt}) = \frac{1}{\sqrt{3}} = \frac{h}{d}
\]

- The hypotenuse is

\[
d = \frac{4}{\sin(\theta_{opt})} \quad \text{with} \quad \sin(\theta_{opt}) = \sqrt{\frac{2}{3}}
\]

- It follows that

\[
h = \frac{4}{\sqrt{2}}
\]
An open box with its base having a length twice its width is to be constructed with 800 square cm of material. **Find its dimensions that maximize the volume.**
An open box with its base having a length twice its width is to be constructed with 800 square cm of material. **Find its dimensions that maximize the volume.**

Let its width be denoted \(x\) and its height be denoted \(y\), then the volume, \(V(x, y)\), of this open box satisfies:

A. \(V(x, y) = 2x^2 + 6xy\)

B. \(V(x, y) = 2x^2y\)

C. \(V(x, y) = 2xy^2\)

D. \(V(x, y) = x^2y\)

E. \(V(x, y) = x^2 + 4xy\)
An open box with its base having a length twice its width is to be constructed with 800 square cm of material. Find its dimensions that maximize the volume.

Let its width be denoted \( x \) and its height be denoted \( y \), then the volume, \( V(x, y) \), of this open box satisfies:

A. \( V(x, y) = 2x^2 + 6xy \)

B. \( V(x, y) = 2x^2y \)

C. \( V(x, y) = 2xy^2 \)

D. \( V(x, y) = x^2y \)

E. \( V(x, y) = x^2 + 4xy \)

This is the Objective function.
The open box is to be constructed with 800 square cm of material. **Find an equation for the surface area, \( S(x, y) \).**
The open box is to be constructed with 800 square cm of material. **Find an equation for the surface area,** $S(x, y)$.

A. $S(x, y) = 2x^2 + 4xy = 800$

B. $S(x, y) = 2x^2y + 2x^2 = 800$

C. $S(x, y) = 2xy^2 = 800$

D. $S(x, y) = 2x^2 + 6xy = 800$

E. $S(x, y) = x^2 + 4xy = 800$
The open box is to be constructed with 800 square cm of material. **Find an equation for the surface area,** $S(x, y)$.

A. $S(x, y) = 2x^2 + 4xy = 800$

B. $S(x, y) = 2x^2y + 2x^2 = 800$

C. $S(x, y) = 2xy^2 = 800$

D. $S(x, y) = 2x^2 + 6xy = 800$

E. $S(x, y) = x^2 + 4xy = 800$

This is the Constraint condition.
Objective function is

\[ V(x, y) = 2x^2 y \]

with Constraint condition

\[ S(x, y) = 2x^2 + 6xy = 800 \]
Objective function is

\[ V(x, y) = 2x^2y \]

with Constraint condition

\[ S(x, y) = 2x^2 + 6xy = 800 \]

Thus, \(6xy = 800 - 2x^2\), or
Objective function is

\[ V(x, y) = 2x^2 y \]

with Constraint condition

\[ S(x, y) = 2x^2 + 6xy = 800 \]

Thus, \( 6xy = 800 - 2x^2 \), or

\[ y = \frac{400}{3x} - \frac{x}{3} \]
Box Problem

**Objective function** is

\[ V(x, y) = 2x^2 y \]

with **Constraint condition**

\[ S(x, y) = 2x^2 + 6xy = 800 \]

Thus, \(6xy = 800 - 2x^2\), or

\[ y = \frac{400}{3x} - \frac{x}{3} \]

The objective function becomes

\[ V(x) = 2x^2 \left( \frac{400}{3x} - \frac{x}{3} \right) = \frac{2}{3}(400x - x^3) \]
Since the objective function is

\[ V(x) = \frac{2}{3}(400x - x^3), \]

we differentiate to obtain
Since the objective function is

\[ V(x) = \frac{2}{3}(400x - x^3), \]

we differentiate to obtain

\[ V'(x) = \frac{2}{3}(400 - 3x^2) \]
Since the objective function is

\[ V(x) = \frac{2}{3}(400x - x^3), \]

we differentiate to obtain

\[ V'(x) = \frac{2}{3}(400 - 3x^2) = 0 \]

for optimal solution.
Since the objective function is

\[ V(x) = \frac{2}{3} (400x - x^3), \]

we differentiate to obtain

\[ V'(x) = \frac{2}{3} (400 - 3x^2) = 0 \]

for optimal solution.

It follows that

\[ x_{opt} = \frac{20}{\sqrt{3}} \]
Since the objective function is
\[ V(x) = \frac{2}{3} (400x - x^3), \]
we differentiate to obtain
\[ V'(x) = \frac{2}{3} (400 - 3x^2) = 0 \]
for optimal solution. It follows that
\[ x_{opt} = \frac{20}{\sqrt{3}} \]
The length, height, and volume are easily obtained from this.