Calculus for the Life Sciences

Lecture Notes – Introduction

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Outline

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TA Contact Information

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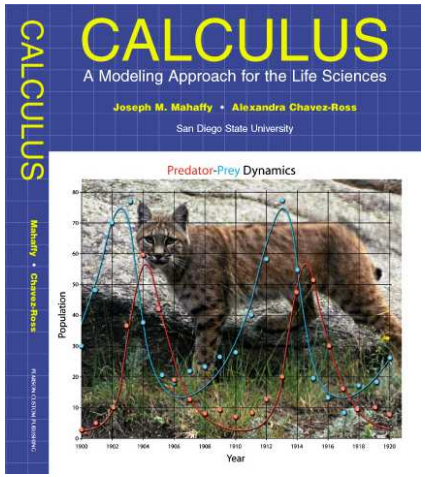
Office Hours	TBA in GMCS 421, and by appointment

Basic Information: The Book



Title:
"Calculus: A Modeling Approach for the Life Sciences"
Volumes I and II

Authors:
Joseph M. Mahaffy &
Alexandra Chàvez-Ross

Publisher:
Pearson Custom Publishing

ISBN:
0-558-17036-6
0-536-90522-3

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Basic Information: Syllabus

- Functions and Models
- Discrete Dynamical Models
- The Derivative
- Optimization
- Differential Equations and Integration

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Basic Information: Grading

Detailed information is found on the
Homework and Assignment Web Page

- Lecture Material is 70% of grade
  - Lecture Participation, i-clicker (7% of Lecture grade)
  - Homework with WeBWorK (9% of Lecture grade)
  - Weekly HW Quizzes, 3 Exams, and Final (84%)
    - Scientific Calculator (TI-30X) only - HW Quizzes, Exams, and Final
    - **1** 3x5 notecard for HW Quizzes and Exams and **3** 3x5 notecards for Final
    - No cell phone visible (ZERO on Exam)
  - Lab Work is 30% of grade
    - 13-14 Lab assignments
    - 3 Lab Exams worth twice a regular Lab assignment
    - Open notes, Computer (except email), No cell phone

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Expectations and Procedures, I

- Lecture class attendance is required for the participation part of the grade — Homework and announcements will be posted on the class web page. If/when you attend class:
  - Please be on time.
  - Please pay attention.
  - Please turn off mobile phones.
  - Please be courteous to other students and the instructor.
  - Abide by university statutes, and all applicable local, state, and federal laws.

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Expectations and Procedures, II

- WeBWorK assignments are posted with a specific due date. It is **your responsibility** to complete the assignment on time.
- The instructor will make special arrangements for students with documented learning disabilities and will `try` to make accommodations for other unforeseen circumstances, *e.g.* illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. ***Please contact the instructor EARLY regarding special circumstances.***
- Students are expected ***and encouraged*** to ask questions in class!
- Students are expected ***and encouraged*** to to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!

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Expectations and Procedures, III

- **Missed Exams or Lab Exams: Don't miss Exams!** You will receive a **ZERO** for any missed exam, except for **written/documented** excuses (illness, personal/family crises, etc.).
- **Lab assignments**:
  - Attendance is mandatory or automatic 10 point deduction
  - Partners are assigned and must work with given partner
  - Arriving 20 minutes late or missing a Lab means working the lab alone
  - Labs due promptly by Thursday 9 PM following a given Lab unless told otherwise.
  - Lowest lab score is dropped
  - Your responsibility to back up Lab work – No excuses accepted or extensions granted for lost material

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Computer Lab

- Computer Labs are located in GMCS 421, 422 and 425 – Hours are posted on the Lab doors
- Completed Lab Reports are turned into Math 124 box located in GMCS 421/422
- Software used
  - Excel
  - Word
  - Maple
- Labs are 70% WeBWorK and 30% written report
- **Please direct questions first to your Lab TA**

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Math 124: Formal Prerequisites

- Good knowledge of High School Algebra
- Score of 74 or higher on ALEKS exam
- Score of 3 or higher on AP Calculus exam
- Grade of C or higher in Math 141

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Math 124: Technology

Design of Calculus for Life Sciences - Math 124

- Lecture and HW
  - Work problems with only Scientific Calculator (TI-30X)
  - Primarily, developing skills through mental activity
  - See **Math Learning** web page (Miscellaneous)
- Computer Lab
  - Manage more complex (realistic) problems and data fitting
  - Use computer programs to help solve these problems
  - Computer Lab problems are linked to Lecture material

No Wolfram Alpha

Students find that **Wolfram Alpha** easily manages their HW
Using this for anything but **checking answers** often results in failing the course

Math 124: Introduction

- Biology is rapidly expanding - more quantitative analysis of the data
- Mathematics and computers are more important
- This course in Calculus for Biology
  - Emphasis on mathematical modeling of biological systems
  - Lecture notes show how Calculus naturally arises in biological examples
  - Begin with a biological model
  - Mathematical theory required to analyze the biological problem
- Use real or realistic examples
- Computer labs aid the more complicated models

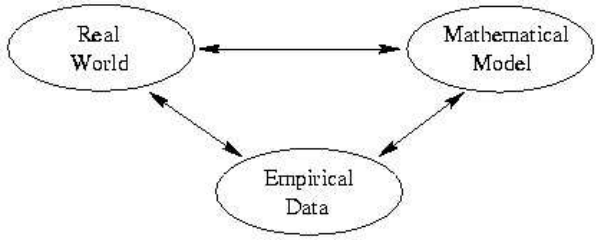
Math 124: Introduction — Mathematical Biology

Mathematical Biology

- Mathematical tools
  - Better qualitative and quantitative understanding of biological problems
  - Suggest alternate possibilities
  - Reject inconsistent ideas
- Biological problems
  - Often stretch mathematical techniques
  - Illustrate mathematical tools well
  - Build intuition for problem techniques

Math 124: Introduction — Mathematical Model

So what is a mathematical model?



Math 124: Introduction — Mathematical Model

- A **mathematical model** is a representation of a real system
- It is simple in design
- It exhibits the basic properties of the real system
- The model should be testable against empirical data
- Comparisons of the model to the real system should lead to improved mathematical models
- The model may suggest improved experiments
- Often there is not an exact answer, differing from K-12 training in mathematics

Introduction – Example – Diabetes mellitus

Biological Information

- Metabolic disease characterized by too much sugar in the blood and urine
- β-cells in the pancreas release insulin in response to rises in levels of glucose in the blood
- Stores energy as glycogen in the liver
- Juvenile diabetes (Type I) - failure of the β-cells to release insulin to blood glucose levels – an autoimmune response killing β-cells
- Adult onset diabetes (Type II) results in insulin resistance – cells fail to use insulin properly

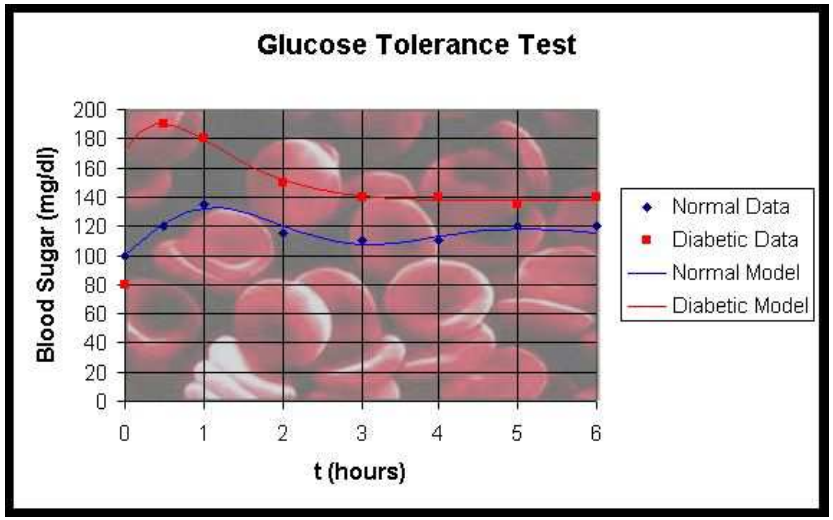
Diabetes mellitus – Ackerman Model

Ackerman Model for Diabetes

- Glucose Tolerance Test (GTT)
  - Subject fasts for 12 hours
  - Given a large quantity of glucose
  - Blood sampled regularly for 4-6 hours
- Mathematical Model
  - 2-Component model - Blood glucose and insulin levels
  - Linear system of differential equations (Damped harmonic oscillator)
  - Simple solution with exponentials and trig functions
  - Solution fit to data
  - Parameter values indicate health of subject

Ackerman Model for Diabetes

Glucose Tolerance Test



Introduction – Example – Predator-Prey Model 1

Predator-Prey Model

Thanks to Tom and Pat Leeson

Example – Predator-Prey Model 2

Predator-Prey Model

- In the early 20^{th} century, Sir Ronald Ross used mathematical modeling to show that malaria could be eliminated without the total eradication of mosquitoes
- A. J. Lotka [1] first studied the population dynamics of predator-prey interactions
- Studies of Vito Volterra on fishing in the Adriatic Sea in 1924 showed value of a simple model for equilibrium analysis
- **Predator-Prey models** are often called **Lotka-Volterra models**
- Widely used by biologists – however, significant flaws in the mathematical understanding often lead to poor conclusions

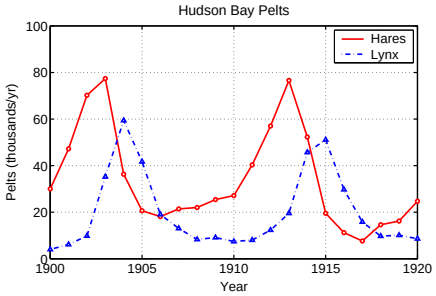
[1] A. J. Lotka (1912), Evolution in Discontinuous Systems, *Journal of the Washington Academy of Sciences*, **2**, pp.2, 49, 66

Example – Predator-Prey Model 3

Classic Lynx-Hare Data

- Records of the Hudson Bay Company show that the pelts of the lynx and hares seemed to oscillate with a fairly regular period
- Simple ecological system, as the lynx is a very specialized predator that primarily feeds on snowshoe hares
- Books often cherry-pick to show limited data - Model fails badly over the complete data set
- We'll examine this model late in the semester

Example – Predator-Prey Model



- Graph shows a clear correlation between the populations of lynx and hares
- Rapid rise in the population of the hares is followed by a rapid rise in the lynx population
- Next the hare population plummets, which is followed by lynx population plummeting

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