1. (1 pt) mathbioLibrary/setABiocLabs/Lab121_C3_yeast_growth.pg

Because of the accuracy of WebWork, you should use 5 or 6 significant figures on all problems.

In 1913, Carlson [1] studied the growth of a culture of yeast, *Saccharomyces cerevisiae*. Over time this culture levels off, but its initial growth is exponential or Malthusian. A Malthusian growth model is given by

\[ P_{n+1} = P_n + rP_n. \]

(We will study this model shortly.) Simply put, the population at the next time \( n + 1 \) is equal to the population at the current time \( P_n \) plus some growth term, which is simply proportional \( (r) \) to the current population. Thus, we have a growth function

\[ g(P) = rP. \]

Below is a table from Carlson’s data showing the population and the rate of growth at that particular population

<table>
<thead>
<tr>
<th>Population ( P )</th>
<th>( g(P) ) Growth/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>19100</td>
<td>10900</td>
</tr>
<tr>
<td>29300</td>
<td>18300</td>
</tr>
<tr>
<td>47300</td>
<td>24300</td>
</tr>
<tr>
<td>71800</td>
<td>48100</td>
</tr>
</tbody>
</table>

a. Use Excel’s Trendline on the data points to find the best straight line passing through the origin. (Note you will need to use the option in trendline of setting the y-intercept = 0.) What is the slope of the line that best fits through the data? Slope \( r = \ldots \).

Find the sum of square errors with this model and the data. Sum of Square Errors = \ldots.

b. In your Lab report create a graph of the data and the best fitting linear model. Briefly describe how well the line fits the data.

c. In lecture (Function Review and Quadratics), we examined a linear model for mRNA synthesis. For the linear model (passing through the origin) given above, we can readily find the sum of squares function. Consider a data point \( (P_i, g(P_i)) \). The absolute error between this data point and our model is given by

\[ e_i = |g(P_i) - rP_i|. \]

Thus, \( e_1 = |10900 - 19100r| \). Similarly, you can find \( e_2, e_3, \) and \( e_4 \). The sum of squares function is given by

\[ J(r) = e_1^2 + e_2^2 + e_3^2 + e_4^2. \]

Find the expression for the quadratic function of the slope of the model, \( r \) (in simplest form).

\[ J(r) = \ldots r^2 + \ldots r + \ldots. \]

Find the coordinates of the vertex.

\[ (r_v, J(r_v)) = (\ldots, \ldots) \]

d. In your Lab report create a graph of \( J(r) \) for \( r \in [0.3, 0.8] \). Compare the value of the vertex of the parabola, \( r_v \), and the slope of the best fitting line through the origin found by Excel’s Trendline. Also, compare the sum of square errors and the value of \( J(r_v) \).

e. From the best model, find the growth for a population of 100,000 yeast.

Growth for 100,000 yeast \( g(100,000) = \ldots \).

Determine the population of another culture of yeast given that their growth rate is measured to be 75,000 yeast/hour.

For \( g(P) = 75,000, P = \ldots \).