1. a. Perform two Monte Carlo simulations for the Schmitz and Kwak study of the Deaconesse Hospital, using their assumptions, for each of the cases with 4 and 5 operating rooms and having a patient load of 32 patients per day (which would be the case load with Sundays and some holidays off). You can simply use two sets of random numbers and apply the same set to both the 4 and 5 operating room cases. Determine the latest times that the operating rooms and recovery room are open. Find the average length of time that the operating rooms are open each day.

b. If you assume that the recovery room must be staffed at all times by at least 2 nurses and that each nurse can handle at most 3 patients, then if the nurses only work integer numbers of hours. Find the number of nursing hours that are required to staff the recovery room according to your simulations.

c. (Bonus) Run the simulations for 4 and 5 operating rooms 100 times and answer the questions above. Give the mean longest time for both the operating and recovery rooms, mean average time of operating rooms being used, and the mean number of hours that are needed for the nursing staff in the recovery room. Also, compute the standard deviation of all of this information.

2. a. Consider a Malthusian growth model with a 4% annual growth:

$$P_{n+1} = 1.04 P_n, \qquad P_0 = 50,$$

where n is in years. Determine the population after 10 years. Find the length of time for this population to double.

b. Now consider a birth only model for this population. Start with a population of 50 individuals. Each individual has a 4% chance of producing an offspring each year. Perform a Monte Carlo simulation for 10 years and give the population for each of the 10 years. Note that new individuals can give birth in subsequent years.

c. Run the simulation in Part b 1000 times and compute the average population at 10 years for these simulations. Also, compute the standard deviation for these 1000 simulations at 10 years. Compare these results to your calculations in Part a.

3. Below is a picture of a standard dart board. Assume that the dartboard has a radius of 12 inches. Assume that starting from the center, the bull's eye (red) has a radius of 0.5 inch and is worth 50 points and the next annular region (green) has a radius of 0.5 inch and is worth 25 points. The black and white sectors are worth the number of points labeled on the outside ring, ranging from 1 to 20 points. The inner of these black and white sectors is an annulus with a radius of 4 inches, and the outer black and white annulus has a radius of 3 inches. The ring between these two annuli (green and red) is a triple score region (three times the score listed in the outside region) and has a radius of 0.5 inches. The outer ring of green and red regions is a double score region with a radius of 0.5 inches. The outer most region has a radius of 3 inches and is worth no points. We assume that each of the 20 sectors has equal area. You might note that the arrangement of the sectors has low value regions next to high value regions, adding significantly to the skill required for hitting the region desired. You may want to learn

the various versions of the game.

a. A standard round of darts has the person throw three darts. Assume that all darts hit the dart board randomly. What is the highest possible score, and how is that achieved? Design a Monte Carlo simulation for scoring three darts with the assumption of hitting the board at random. List 5 sets of three darts thrown giving the score of each dart and the total for each of the 5 rounds.

b. Simulate your program 10,000 times to obtain the mean score of throwing three darts and the standard deviation of the score for your 10,000 rounds of three darts.

c. In fact, someone throwing darts has an objective of winning the game, and different players have differing abilities. Write a brief discussion on how you might modify your Monte Carlo simulation to better simulate a real game conditions. (You might want to look up the rules for winning at darts to answer this.)

4. In lecture, we saw that a stochastic birth only process could be given by the differential equation

$$\frac{dP_n}{dt} + \lambda N P_n = \lambda(N-1)P_{N-1},$$

with

$$P_N(0) = \begin{cases} 0 & N \neq N_0 \\ 1 & N = N_0 \end{cases},$$

where $P_N(t)$ is the probability that there are exactly N individuals in a population. The solution was given by

$$\begin{aligned} P_{N_0}(t) &= e^{-\lambda N_0 t} \\ P_{N_0+1}(t) &= N_0 e^{-\lambda N_0 t}(1-e^{-\lambda t}) \\ &\dots \\ P_{N_0+j}(t) &= \frac{N_0(N_0+1)\cdots(N_0+j-1)}{j!} e^{-\lambda N_0 t}(1-e^{-\lambda t})^j. \end{aligned}$$

Use mathematical induction to prove that this last formula holds for all j.

5. The expected population at time t, $E(t)$, is obtained from the formula

$$E(t) = \sum_{j=0}^{\infty} (N_0 + j) P_{N_0+j}(t).$$

a. Can you explain why this is valid using probabilistic ideas?

b. Show that $E(t) = N_0 e^{\lambda t}$. [Hint: Explicitly calculate the first few terms of the summation; compare this to the derivative of the binomial expansion.]

c. Explain the significance of Part b.

6. Consider the equation above for $P_{N_0+j}(t)$. Assume that $N_0 = 10,000$. Estimate λ, if it is observed that a total of 4500 births occur in 20 days. [Hint: See the previous exercise.]