1. a. The population of Canada was 24,070,000 in 1980, while in 1990 it was 26,620,000. Assuming the population is growing according to the principle of the continuous Malthusian growth (with no food or space limitations), find the population as a function of time, and determine its doubling time.

b. For the same years, the populations of Kenya were 16,681,000 and 24,229,000, respectively. Find the population of Kenya as a function of time, assuming it too is growing with Malthusian growth. What is Kenya's doubling time for its population?

c. Use these models to project the populations in the two countries in the year 2000. In what year do the populations of Canada and Kenya become equal?

2. An older woman is quite ill, and her daughter finds that she has been running a temperature of 39°C. Over the night, the woman passes away in her sleep, and the daughter discovers her death at 7 AM. At this time the body is found to be 35°C. Two hours later the body temperature is 33.5°C. The woman's bedroom maintained a temperature of 25°C. If the body satisfies Newton's law of cooling,

$$\frac{dH(t)}{dt} = -k(H(t) - T_e),$$

where T_e is the temperature of the bedroom, t is in hours, H is the temperature in °C, and k is the coefficient of heat transfer to be determined (to **4 significant figures**) for this woman. Determine when the woman died (using normal time, hours and minutes).

3. The data in the following table were obtained by J. M. Cushing using a kitchen thermometer heated in an oven to 150°F and then observed to cool. Use Newton's law of cooling to fit

t (min)	0	1	2	3	4	5	6
Temp (°F)	150	109	94	85	83	81	80

these temperatures to the best least squares model for the data. Allow adjustment of initial temperature, rate of cooling, and room temperature (fixed) to vary. Include your sum of square errors between the model and the data. Find the percent error between the best fitting model and the temperature recorded at $t = 5$.

4. (The Allee effect) For higher organisms, the growth rate is more complicated than the logistic growth model. For example, reproduction could be reduced if it becomes too difficult to find a mate (which is one problem apparently facing the Giant Panda). An alternate higher order model to the logistic growth model is one modeling the Allee effect. A differential equation for this model is given by

$$\frac{dP}{dt} = P\left(r - a(P-b)^2\right),$$

where $b > \sqrt{r/a}$ and all parameters are positive.

a. Find all equilibria and determine the stability of the equilibria. Draw a phase portrait of this model.

b. Compare this model to the logistic growth model. Describe the similarities and differences between these models. Write a brief paragraph discussing how the equation above would relate to some animal population, *i.e.*, give a brief ecological interpretation of the model.

5. In this problem you will repeat much of the work that was done in the lecture notes on two competing yeast populations, only with some graminivorous beetles. In this problem, you will take the data from A. C. Crombie [1] on the beetles, *Rhizopertha dominica*, the lesser grain borer, and *Oryzaephilus surinamensis*, the saw-tooth grain beetle. You can find the data in an Excel file from my webpage. The first two columns give the data for an experiment with only *Rhizopertha*. The columns D and E give the data for *Oryzaephilus* growing alone. Finally, columns G, H, and I show the results of an experiment with both species growing together.

a. Use the data with each species growing alone to find the best continuous Malthusian growth model for *Rhizopertha dominica* using the first 119 days of data. Then repeat this process for *Oryzaephilus surinamensis*, using the first 77 days of data. Give all of the parameters that you find and show graphs of both the data and the models. Write the solutions to your model and give the sum of squares error between the model and the data.

b. Next, take the data (all values) with each species growing alone to find the best fit to a logistic growth model for each of the beetle species. Once again, give all of the parameters in the models, write the solutions of the models, and show graphs of both the data and the models. List the sum of squares error between the model and the data.

c. Use the information from Part b to fix the Malthusian growth parameter and the intraspecies competition term. A quasi-steady state analysis will allow you to obtain estimates for the interspecies competition terms. (It turns out that you need to begin with the initial population guess for *Rhizopertha* near 5 and the initial guess for *Oryzaephilus* near 0.5.) Use the MatLab program to find the best fit to the data from the experiment with both species growing together. Write the sum of squares error between the model and the data. (I do NOT believe that the Excel program will work.) Give all of the parameters that you find and show graphs of both the data and the models.

d. For the model that you found in Part c, find all equilibria and find the eigenvalues at those equilibria. Graph the nullclines, clearly labeling the equilibria. Show the direction field for this system of differential equations (being sure that the nullclines are clearly visible). Discuss the stability of each of the equilibria and predict what will happen with the populations of these beetles over a long period of time, assuming the experimental conditions hold.

[1] A. C. Crombie (1945). On competition between various species of graminivorous insects, *Proc. Roy. Soc* (B) **132**, 362-395.

6. Consider the general competition model given by the equations:

$$\begin{aligned}\frac{dx}{dt} &= x(a_1 - a_2x - a_3y),\\ \frac{dy}{dt} &= y(b_1 - b_2y - b_3x).\end{aligned}$$

Sketch the phase plane, perform a nullcline analysis, and draw representative trajectories of both populations if

a. $\frac{b_1}{b_2} > \frac{a_1}{a_3}$ and $\frac{b_1}{b_3} > \frac{a_1}{a_2}$.

b. $\frac{a_1}{a_3} > \frac{b_1}{b_2}$ and $\frac{b_1}{b_3} > \frac{a_1}{a_2}$.

Describe the interaction between the two species in each case. Clearly, state the relative strengths of intra and interspecies competition between the species as related to the inequalities. State all biologically feasible equilibria for each case, then describe what type of equilibrium point each one is (saddle node, unstable node, etc.).

7. Consider the model given by the equations:

$$\begin{aligned}\frac{dX}{dt} &= X(2 - 2X + Y),\\ \frac{dY}{dt} &= Y(1 - Y + X).\end{aligned}$$

a. Give a brief explanation of each species' ecological behavior. Describe each term on the right hand side of the differential equations.

b. Determine all possible equilibria.

c. In the phase plane, draw the nullclines. Introduce arrows to show representative directions of the trajectories.

d. Perform a linear stability analysis, giving eigenvalues and eigenvectors at each equilibrium. Classify the equilibria (*e.g.*, stable node).

e. Discuss what happens ultimately for this biological system.