Homework 1

Burden-Faires: 1.1.16, 1.2.4, 1.2.9, 1.3.2, 1.3.7

1. The Greek mathematician Archimedes estimated the number π by approximating the circumference of a circle of diameter 1 by the perimeter of both inscribed and circumscribed polygons. The perimeter s_n of the inscribed regular polygon with $2n$ sides and t_n the circumscribed regular polygon with $2n$ sides can be given recursively by the following formulas:

$$s_n < \pi < t_n,$$

where

$$s_{n+1} = \frac{2^{n+1}}{\sqrt{2}} \sqrt{1 - \sqrt{1 - \left(\frac{s_n}{2^n}\right)^2}}, \qquad s_2 = 2\sqrt{2}$$

and

$$t_{n+1} = \frac{2^{n+1} \left(\sqrt{1 + \left(\frac{t_n}{2^n}\right)^2} - 1 \right)}{\frac{t_n}{2^n}}, \qquad t_2 = 4$$

(a) Calculate s_3 to s_{30}.

(b) Calculate t_3 to t_{30}.

(c) What went wrong with the calculation (a) and (b)?

(d) Correct the problem and recompute s_3 to s_{30} and t_3 to t_{30}.

2. Use the bisection method to find the root of

$$f(x) = x \cdot \cos(x) - \ln(x + 1)$$

in the interval $[0.2, 1.6]$ with an error less than 10^{-5}. How many iterations are required with the bisection method for the desired accuracy?