1. Currently there is a debate on the importance of preserving large tracts of land to maintain biodiversity. Many of the arguments for setting aside large tracts are based on studies of biodiversity on islands. This problem uses allometric modeling to determine the number of species of herpetofauna (amphibians and reptiles) as a function of island area for the given Caribbean islands. You are given the following data [1]:

Island	Area (mi^2)	Species
Redunda	1	3
Montserrat	33	10
Jamaica	4,411	38
Cuba	46,736	97

a. Let N be the number of species and A be the area of the island, then the allometric relation for the number of species to the area of the island is given by

$$N = kA^a.$$

Find the best power law fit to the data and graph the data and the model. How well does the graph match the data?

b. From the model above give estimates to fill in the table below.

Island	Area (mi^2)	Species
Saba	5	
Puerto Rico		40
Saint Croix	80	
Hispaniola		88

How important is maintaining a large tract of land to the maintenance of biodiversity based on this model? What does the model predict is required in increased preserved area to double the number of species supported by the environment? (Give a numerical value for the factor multiplying the area of land to achieve this doubling of species.)

2. The data below came from the Allegheny National Forest in Pennsylvania. Various trees were measured by diameter (in inches), height (in ft) and volume (in board feet). You want to find the volume as a function of either diameter or height and determine a meaningful relationship.

a. The reference for this model suggests a simple linear model, so find the best linear model through the data. Graph the data and model for volume as a function of diameter, then repeat the process for volume as a function of height. Give the formula for the best straight line through each of the data sets. Which graph seems to have the better predictive ability? Why is this what you would expect based on the biology of trees? What happens with both models as the diameter or height gets close to zero?

[1] Data from J. Mazumdar, *An Introduction to Mathematical Physiology and Biology*, Cambridge, 1989.

Diameter	Height	Volume
8.6	65	10.3
10.7	81	18.8
11	75	18.2
11.4	76	21.4
12	75	19.1
13.3	86	27.4
14.5	74	36.3
16	72	38.3
17.3	81	55.4
18	80	51.5
20.6	87	77

b. In this part of the problem only use the relationship between volume and the one variable that you showed in Part a was the better predictor. Find an allometric model that best fits the one set of data that best predicts the volume. Plot the data and the best power law fit on a graph. Provide an explanation for the power that you have obtained.

3. a. This problem relates to Kepler's Third Law. Use the power law to determine the period of revolution around sun based on the distance from the sun for all planets given information about some of the planets. Let d be the mean distance ($\times 10^6$ km) from the sun and p be the period of revolution in days about the sun. You are given the following data [2] concerning four of the planets:

Planet	Distance	Period p
	d ($\times 10^6$ km)	days
Mercury	57.9	87.96
Earth	149.6	365.25
Mars	227.9	687.0
Jupiter	778.3	4337

a. The power law expression relating the period of revolution (p) to the distance from the sun (d) is given by

$$p = kd^a,$$

where k and a are constants to be determined. Use this power law to best fit the data above.

b. Use the power law found in Part a to complete the table below:

c. Use dimensional analysis to prove that the power you obtained in Part a is the correct value in Kepler's Third Law. You should use that the gravitational force

$$F_g = \frac{GMm}{d^2},$$

balances the centripetal force

$$F_c = \frac{mv^2}{d},$$

[2] Jay M. Pasachof, (1993) *Astronomy: From the Earth to the Universe*, Fourth Edition, Saunders College Publishing.

Planet	Distance	Period p
	d ($\times 10^6$ km)	days
Venus	108.2	
Saturn		10,760
Uranus	2871	
Neptune	4497	

where G is the universal gravitation constant, M is the mass of the sun, m is the mass of the planet, d is the same as above, and v is the velocity of the planet.

4. a. It can be shown that puppies put on weight (in kg) in a manner that fits the function

$$W(t) = \frac{W_0 M}{W_0 + (M - W_0)e^{-rt}},$$

where W_0 represents the birth weight of the puppy, M is the final weight, r is a growth rate, W is the weight in kg, and t is the age in days. Below is a table for the growth of a Golden Retriever puppy.

t (day)	Weight (kg)
0	1.5
20	2.5
40	4.1
60	6.8
100	13.6
150	24.5
200	29.5
250	32.0
350	34.2
450	35

a. Find the best values of the parameters W_0, M, and r that fit the data.

b. The ASPCA Dog Care Manual gives the following table for the amount of calories that a dog needs for a given weight:

Weight (kg)	Calories
5	450
10	750
20	1250
30	1700
40	2100
50	2500

Find the best allometric model of the form

$$C(W) = kW^a,$$

where k and a are constants that fits the data above.

c. Create a composite function to give the amount of calories needed as a function of age from the two models above, *i.e.*, find $C(t)$. Determine when the rate of change in the amount of calories is at a maximum. Give the age, t, when this occurs, as well as the number of calories required $C(t)$ and rate of change in the amount of calories $C'(t)$ at this time.

5. The Head of the Charles rowing race in Boston is a 3.5 mile autumn race on the Charles river. You are given the following data and want to perform a similar analysis to the one given in class on rowing. The winning times for the eights are about 16 minutes, the fours are 18.5 minutes, and the singles are 24 minutes. Find the best allometric model and explain why this race doesn't fit our dimensional analysis model very well.

6. In Jonathan Swift's *Gulliver's Travels*, the Lilliputians decided to feed Gulliver 1728 times as much food as a Lilliputian ate. They reason that since Gulliver was 12 times their height, his volume was $12^3 = 1728$ times the volume of a Lilliputian. Why is their reasoning wrong and what is the correct answer?

7. Suppose that a certain windmill on average produces 10 kilowatts of electricity at a given test site. Now we move the windmill to a higher elevation where the air is is dense but the wind blows harder. At this new location, the air is 10% less dense and the wind blows with an average velocity that is 25% faster. What will be the average output of electricity at this new location?