1. (The Allee effect) For higher organisms, the growth rate is more complicated than the logistic growth model. For example, reproduction could be reduced if it becomes too difficult to find a mate (which is one problem apparently facing the Giant Panda). An alternate higher order model to the logistic growth model is one modeling the Allee effect. A differential equation for this model is given by

$$\frac{dP}{dt} = P\left(r - a(P-b)^2\right),$$

where $b > \sqrt{r/a}$.

a. Find all equilibria and determine the stability of the equilibria. Draw a phase portrait of this model.

b. Compare this model to the logistic growth model. Describe the similarities and differences between these models. Write a brief paragraph discussing how the equation above would relate to some animal population, *i.e.*, give a brief ecological interpretation of the model.

2. Consider the two-species competition model given by

$$\begin{aligned} \dot{x}_1 &= x_1(a - bx_1 - kx_2) \\ \dot{x}_2 &= x_2(c - dx_2 - \sigma x_1) \end{aligned}$$

a. Give an ecological interpretation to the quantities

1. $x_1 = a/b$ and $x_1 = c/\sigma$
2. $x_2 = a/k$ and $x_2 = c/d$.

b. Only using a rough phase plane analysis, show that the coexistent equilibrium population is unstable if $a/b > c/\sigma$ and $c/d > a/k$ and stable if $a/b < c/\sigma$ and $c/d < a/k$. Briefly explain this result using the terminology introduced in Part a.

3. Consider the two-species competition model given by

$$\begin{aligned} \dot{F} &= F(2 - 2F + G) \\ \dot{G} &= G(1 - G + F) \end{aligned}$$

a. Give a brief explanation of each species' ecological behavior. (Account for each term on the right-hand side of the above differential equation.)

b. Determine all possible equilibrium populations.

c. In the phase space, draw nullclines for each of the variables.

d. Introduce arrows indicating the direction of the trajectories of this ecosystem. ($F \geq 0$ and $G \geq 0$.)

e. From the phase plane in Part d, briefly explain which (if any) of the equilibria are stable and which are unstable. Do **not** do a linearized stability analysis.

4. In this problem you will repeat much of the work that was done in the lecture notes on two competing yeast populations, only with some graminivorous beetles. In this problem, you will take the data from A. C. Crombie [1] on the beetles, *Rhizopertha dominica*, the lesser grain borer, and *Oryzaephilus surinamensis*, the saw-tooth grain beetle. You can find the data in an Excel file from my webpage. The first two columns give the data for an experiment with only *Rhizopertha*. The columns D and E give the data for *Oryzaephilus* growing alone. Finally, columns G, H, and I show the results of an experiment with both species growing together.

a. Use the data with each species growing alone to find the best Malthusian growth model for *Rhizopertha dominica* using the first 119 days of data. Then repeat this process for *Oryzaephilus surinamensis*, using the first 77 days of data. Give all of the parameters that you find and show graphs of both the data and the models. Write the solutions to your model and give the sum of squares error between the model and the data.

b. Next, take the data (all values) with each species growing alone to find the best fit to a logistic growth model for each of the beetle species. Once again, give all of the parameters in the models, write the solutions of the models, and show graphs of both the data and the models. List the sum of squares error between the model and the data.

c. Use the information from Part b to fix the Malthusian growth parameter and the intraspecies competition term. A quasi-steady state analysis will allow you to obtain estimates for the interspecies competition terms. (It turns out that you need to begin with the initial population guess for *Rhizopertha* near 5 and the initial guess for *Oryzaephilus* near 0.5.) Use the MatLab program to find the best fit to the data from the experiment with both species growing together. Write the sum of squares error between the model and the data. (I do NOT believe that the Excel program will work.) Give all of the parameters that you find and show graphs of both the data and the models.

d. For the model that you found in Part c, find all equilibria and find the eigenvalues at those equilibria. Graph the nullclines, clearly labeling the equilibria. Show the direction field for this system of differential equations (being sure that the nullclines are clearly visible). Discuss the stability of each of the equilibria and predict what will happen with the populations of these beetles over a long period of time, assuming the experimental conditions hold.

From the text
P.90 - Problems 3.9.1, 2, 3, 4, 6, 7