Due November 12, 2004 Math 342A Name ______________________________

1. Consider the following matrices and perform the operations listed when possible:

$$A = \begin{pmatrix} -1 & 0 & 2 \\ 3 & 3 & 1 \\ 2 & -2 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} 4 & -1 \\ 0 & 2 \\ 1 & 3 \end{pmatrix}, \qquad C = \begin{pmatrix} 3 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

a. Find the rank and determinant of each of the above matrices.

b. Calculate AB, $(AB)^T C$, BA, $2A - B$, and $3A + C$.

c. Find the eigenvalues and a basis of eigenvectors for A and C. Determine the algebraic and geometric multiplicity of each of the eigenvalues.

2. Consider the following matrix:

$$Q = \begin{pmatrix} 4+a & -2-a & 0 & -4 \\ 4 & -2 & 0 & -4 \\ 0 & 3-a & 3 & -3+a \\ 4 & -2-a & 0 & -4+a \end{pmatrix}.$$

a. Find the rank and determinant of Q. Find the eigenvalues and eigenvectors of Q. Be sure to give the algebraic and geometric multiplicity of the eigenvalues (noting all special cases for particular values of a).

b. Consider the system of equations

$$Qx = 0.$$

Find what values of a give non-trivial solutions and find those solutions. What is the solution for $a = 5$?

c. Diagonalize Q, if possible. What values of a result in Q being non-diagonalizable and why?

3. Solve the following system of equations:

$$\begin{array}{rcrcrcrcrcl} x_1 & + & x_2 & + & x_3 & + & x_4 & + & x_5 & = & 4, \\ 2x_1 & + & x_2 & & & & & + & 3x_5 & = & 2, \\ -2x_1 & & & + & 2x_3 & + & x_4 & - & 4x_5 & = & 0, \\ & & x_2 & + & 2x_3 & + & x_4 & - & x_5 & = & 2. \end{array}$$

4. Consider the following matrix:

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & -c \\ b & c & 0 \end{pmatrix}.$$

a. Find the determinant, eigenvalues, and eigenvectors for this matrix.

b. Does the set of matrices of the type above (with a, b, and c arbitrary) form a vector space? If so, find a basis and determine its dimension. If not, why?

5. a. For which values of α will the following system have no solutions? Exactly one solution? Infinitely many solutions?

$$\begin{array}{rcrcrcl} x_1 & + & x_2 & & & = & 3, \\ 2x_1 & - & x_2 & - & 3x_3 & = & 0, \\ 6x_1 & - & 3x_2 & - & \alpha^2 x_3 & = & \alpha + 3. \end{array}$$

b. Solve this system for $\alpha = -3$.

6. a. Form the vector space V of cubic polynomials without any first order terms

$$V = \{p | p(x) = a_3 x^3 + a_2 x^2 + a_0, \text{ for } x \in [0, 1], a_i \in R\}.$$

Find a basis and the dimension of this vector space.

b. Form the subspace W of V with $p(0) = 0$. Find the dimension of this subspace and find a basis.

c. Define an inner product for part b. as follows:

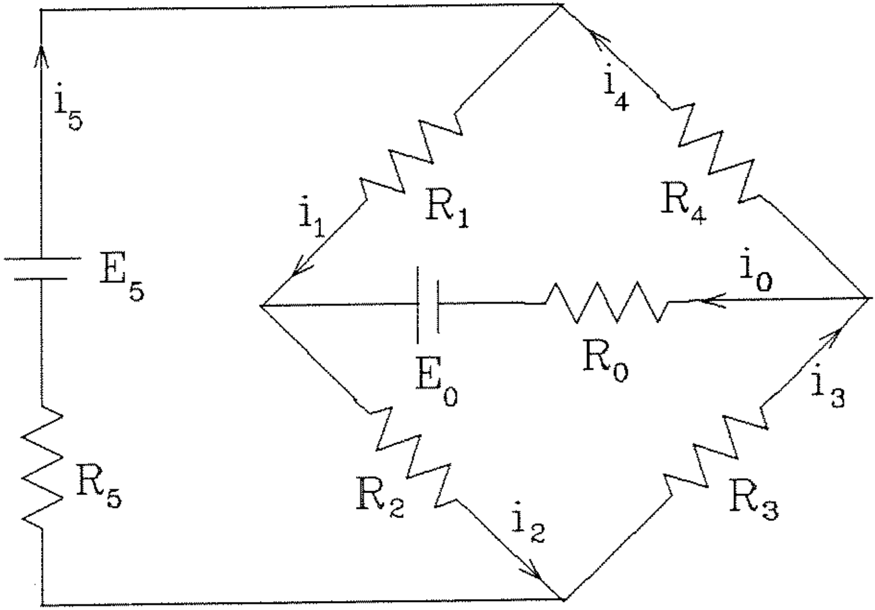
$$\langle f, g \rangle = \int_0^1 f(x) g(x) dx.$$

Clearly, $p_3(x) = x^3$ is an element of W. Find an element in W orthogonal to p_3.

7. The electric circuit presented below has the following resistances (in ohms) and electromotive forces (in volts):

$$R_0 = 15,\ R_1 = 10,\ R_2 = 20,\ R_3 = 5,\ R_4 = 30,\ R_5 = 20,\ E_0 = 50,\ \text{and } E_5 = 100.$$

Use Kirchhoff's laws to determine the unknown currents.



i_5
E_5
R_5
i_4
R_1
R_4
i_1
i_0
E_0
R_0
i_3
R_2
R_3
i_2