

## HW6 9.1 1, 3, 6, 7, 21

$$7. \begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 0 & 0 \\ 1 & 0 & x & 0 \\ 1 & 0 & 0 & x \end{vmatrix} = - \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} + x \begin{vmatrix} x & 1 & 1 \\ 1 & x & 0 \\ 1 & 0 & x \end{vmatrix} = - \begin{vmatrix} x & 0 \\ 0 & x \end{vmatrix} + x \left( - \begin{vmatrix} 1 & 1 \\ 0 & x \end{vmatrix} + x \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} \right)$$

$$= -x^2 + x(-x + x + x(x^2 - 1)) = x^4 - 3x^2$$

$$x = 0, 0, \pm\sqrt{3}$$

$$4 + 5 = 9$$

## HW7 9.2 2, 4, 7, 10

$$7. \left[ \begin{array}{cccc|c} 2 & -4 & 1 & -3 & 6 \\ 1 & -2 & 3 & 4 & 2 \end{array} \right] \xrightarrow[\substack{R_2 - R_1}{\frac{1}{2}R_1}]{\substack{R_2 - R_1 \\ \frac{1}{2}R_1}} \left[ \begin{array}{cccc|c} 1 & -2 & \frac{1}{2} & -\frac{3}{2} & 3 \\ 0 & 0 & \frac{5}{2} & \frac{11}{2} & -1 \end{array} \right] \xrightarrow{\frac{2}{5}R_2} \left[ \begin{array}{cccc|c} 1 & -2 & \frac{1}{2} & -\frac{3}{2} & 3 \\ 0 & 0 & 1 & 3 & -\frac{2}{5} \end{array} \right]$$

$$x_4 = s, \quad x_3 = -3s - \frac{2}{5}, \quad x_2 = t, \quad x_1 = 2t + \frac{1}{2}(3s + \frac{2}{5}) + \frac{5}{2}s + 3$$

$$\therefore x_1 = 2t + 3s + \frac{16}{5}$$

## 9.3 2, 4, 5, 7, 8, 9, 12, 16, 19, 24

$$8. C_3 C_3^T = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_V \sigma_V^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

All orthogonal

$$\sigma_V' (\sigma_V')^T = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_V'' (\sigma_V'')^T = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$16. A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad A^{-1} = \frac{1}{-2} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} \quad (A^{-1})^{-1} = -2 \begin{pmatrix} -\frac{1}{2} & -1 \\ -\frac{3}{2} & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$$

$$(A^T)^{-1} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}^{-1} = \frac{1}{-2} \begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -2 & \frac{3}{2} \\ 1 & -\frac{1}{2} \end{pmatrix} = (A^{-1})^T$$

$$\det(A^{-1}) = -\frac{1}{2} \quad (\det(A))^{-1} = (-2)^{-1} = \det(A^{-1})$$

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 10 & 7 \\ 22 & 15 \end{pmatrix} \quad \det(AB) = -4 \quad \therefore (AB)^{-1} = -\frac{1}{4} \begin{pmatrix} 15 & -7 \\ -22 & 10 \end{pmatrix} \quad B^{-1} = \frac{1}{2} \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$$

$$B^{-1} A^{-1} = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{15}{4} & \frac{7}{4} \\ \frac{11}{2} & -\frac{5}{2} \end{pmatrix} = (AB)^{-1}$$

$$11 + 15 = 26$$