1. Work Problem 6.2 (p. 76) from the text. For the following matrices, compute $\mathbf{A}\mathbf{A}^T$, $\mathbf{A}^T\mathbf{A}$, $(\mathbf{A}^T\mathbf{A})^2$, $(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B})^T$, where

$$\mathbf{A} = \begin{pmatrix} 5 & -3 & 0 \\ 6 & 1 & -4 \end{pmatrix}$$

and

$$\mathbf{B} = \begin{pmatrix} -2 & 4 & -1 \\ 1 & 1 & 5 \end{pmatrix}$$

2. Work Problem 6.7 (p. 76) from the text. If

$$\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad \text{verify that} \qquad \mathbf{A}^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$$

for $n = 2, 3, 4$. What does this mean in terms of rotations through an angle θ? (Use the Maple command map(combine,$\mathbf{A}^2$), etc., which operates on each entry separately. Type ?map for information.)

3. a. Work Problem 6.11 (p. 77) from the text. Verify the basic relation $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & -2 & -1 \\ -2 & 3 & 2 \\ -1 & 2 & 1 \end{pmatrix}$$

and

$$\mathbf{B} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{pmatrix}$$

b. Find the eigenvalues and eigenvectors for both $\mathbf{A}$ and $\mathbf{B}$. Also, find the characteric matrix and characteristic polynomial for $\mathbf{A}$.

4. Solve the following linear system of equations (if a solution exists)

$$\begin{aligned} 2x + y - 3z &= 4 \\ x - y + 2z &= 7 \\ x + 5y - 12z &= -13 \end{aligned}$$

5. Work Problem 7.11 (p. 86) from the text. Show that $\mathbf{A}$ and $\mathbf{B} = \mathbf{P}^{-1}\mathbf{AP}$ have the same eigenvalues and establish the relation between their eigenvectors, where $\mathbf{A}$ and $\mathbf{P}$ are as follows:

$$\mathbf{A} = \begin{pmatrix} -1 & -3 & 3 \\ -6 & 2 & 6 \\ -3 & 3 & 5 \end{pmatrix}$$

and

$$\mathbf{P} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -3 & 2 \end{pmatrix}$$