1. a. With \( n = 2 \) and \( x \in [0, 2] \), the midpoints of the subintervals are \( x_1 = \frac{1}{2} \) and \( x_2 = \frac{3}{2} \) with \( \Delta x = 1 \), so the midpoint rule gives
\[
\int_0^2 (4 + 2x^2)dx \simeq \left( \left( 4 + 2 \left( \frac{1}{2} \right)^2 \right) + \left( 4 + 2 \left( \frac{3}{2} \right)^2 \right) \right) \cdot 1 = 13.
\]
With \( n = 2 \), the trapezoid rule gives
\[
\int_0^2 (4 + 2x^2)dx \simeq \left( \frac{1}{2} (4 + 2(0)^2) + (4 + 2(1)^2) + \frac{1}{2} (4 + 2(2)^2) \right) \cdot 1 = 14.
\]

b. With \( n = 4 \), the subintervals have length \( \Delta x = \frac{1}{2} \), so the midpoint rule gives
\[
\int_0^2 (4 + 2x^2)dx \simeq \left( \left( 4 + 2 \left( \frac{1}{4} \right)^2 \right) + \left( 4 + 2 \left( \frac{3}{4} \right)^2 \right) + \left( 4 + 2 \left( \frac{5}{4} \right)^2 \right) + \left( 4 + 2 \left( \frac{7}{4} \right)^2 \right) \right) \cdot \frac{1}{2} = 13.25.
\]

With \( n = 4 \), the trapezoid rule gives
\[
\int_0^2 (4 + 2x^2)dx \simeq \left( \frac{1}{2} (4 + 2(0)^2) + \left( 4 + 2 \left( \frac{1}{2} \right)^2 \right) + (4 + 2(1)^2) + \left( 4 + 2 \left( \frac{3}{2} \right)^2 \right) + \frac{1}{2} (4 + 2(2)^2) \right) \cdot \frac{1}{2} = 13.5.
\]

c. For \( n = 2 \), the midpoint rule has a \( 100 \left( \frac{13 - 40/3}{40/3} \right) = -2.5\% \) error, which is a low estimate. The trapezoid rule has a \( 100 \left( \frac{14 - 40/3}{40/3} \right) = 5.0\% \) error, which is a high estimate. Similarly, for \( n = 4 \), the midpoint rule has a \(-0.625\% \) error, which is a low estimate. The trapezoid rule has a \( 1.25\% \) error, which is a high estimate.

4. a. Since \( f(0) = 8 \), the \( y \)-intercept is \((0, 8)\). From \( 8 + 2x - x^2 = -(x + 2)(x - 4) = 0 \), the \( x \)-intercepts are \((-2, 0)\) and \((4, 0)\). The midpoint between the \( x \)-intercepts is \( x = 1 \) with \( f(1) = 9 \), so the vertex is \((1, 9)\). See the graph on the other solution page.

b. With \( n = 4 \) and \( x \in [0, 4] \), the midpoints of the subintervals are \( x_1 = \frac{1}{2} \), \( x_2 = \frac{3}{2} \), \( x_3 = \frac{5}{2} \), and \( x_4 = \frac{7}{2} \) with \( \Delta x = 1 \), so the midpoint rule gives
\[
\int_0^4 (8 + 2x - x^2)dx \simeq \left( \left( 8 + 2 \left( \frac{1}{2} \right) - \left( \frac{1}{2} \right)^2 \right) + \left( 8 + 2 \left( \frac{3}{2} \right) - \left( \frac{3}{2} \right)^2 \right) + \left( 8 + 2 \left( \frac{5}{2} \right) - \left( \frac{5}{2} \right)^2 \right) \right) + \left( 8 + 2 \left( \frac{7}{2} \right) - \left( \frac{7}{2} \right)^2 \right) \right) \cdot 1 = 27.0.
\]
With \( n = 4 \), the trapezoid rule gives

\[
\int_0^4 (8 + 2x - x^2)\,dx \approx \left( \frac{1}{2} (8 + 2(0) - (0)^2) + (8 + 2(1) - (1)^2) + (8 + 2(2) - (2)^2) \\
+ (8 + 2(3) - (3)^2) + \frac{1}{2} (8 + 2(4) - (4)^2) \right) \cdot 1 = 26.0.
\]

c. Simpson’s rule gives the exact value with

\[
\int_0^4 (8 + 2x - x^2)\,dx = \left( (8 + 2(0) - (0)^2) + 4 (8 + 2(1) - (1)^2) + 2 (8 + 2(2) - (2)^2) \\
+ 4 (8 + 2(3) - (3)^2) + (8 + 2(4) - (4)^2) \right) \cdot \frac{1}{3} = 26.6667.
\]

For \( n = 4 \), the midpoint rule has a 1.25% error, which is a high estimate. The trapezoid rule has a \(-2.5\%\) error, which is a low estimate.