1. For each of the following differential equations, solve the differential equation. Sketch the graph of the right hand side of the differential equation, then draw the phase portrait.

a. $\dfrac{dy}{dt} = 10 - 0.2\,y,$

b. $\dfrac{dy}{dt} = 0.1\,y - 12.$

2. For each of the following differential equations, find all equilibria. Sketch the graph of the right hand side of the differential equation, then draw the phase portrait.

a. $\dfrac{dy}{dt} = 0.3\,y(4 - y^2),$

b. $\dfrac{dy}{dt} = 0.1y\left(1 - \dfrac{y}{20}\right),$

c. $\dfrac{dy}{dt} = 0.8 - 0.2\,y - 0.1y^2,$

d. $\dfrac{dy}{dt} = -\dfrac{0.2\,y}{1 + y^2},$

e. $\dfrac{dy}{dt} = 0.1y^2 - 4\,y,$

f. $\dfrac{dy}{dt} = \cos(y),$

3. (Subcritical Pitchfork bifurcation) Consider the following differential equation

$$\frac{dy}{dt} = y^3 - \alpha y.$$

Sketch the graph of the right hand side of the differential equation, then draw the phase portrait for $\alpha = 4$ and -4. Find the equilibria for each case and determine their stability. What are the differences in behavior of these two cases? What value of α results in the change between these two behaviors?

4. (Transcritical bifurcation) Consider the following differential equation

$$\frac{dy}{dt} = \alpha y - y^2.$$

Sketch the graph of the right hand side of the differential equation, then draw the phase portrait for $\alpha = 3$ and -3. Find the equilibria for each case and determine their stability. Describe the differences in behavior of these two cases.

5. (Saddle-node or Blue sky bifurcation) Consider the differential equation

$$\frac{dy}{dt} = \alpha - y^2.$$

Sketch the graph of the right hand side of the differential equation, then draw the phase portrait for $\alpha = 4$ and -4. Find the equilibria for each case and determine their stability. What are the differences in behavior of these two cases? What value of α results in the change between these two behaviors?

6. (Allee effect) Suppose that a population, $P(t)$ (in thousands), is given by the model

$$\frac{dP}{dt} = P\left(4 - 0.01(P - 50)^2\right).$$

Sketch a graph of the right hand side of the differential equation, then draw the phase portrait. Find any equilibria and determine their stability. Find the carrying capacity for this particular population. Determine the critical threshhold number of animals required to avoid extinction.

7. (Harvesting 1) Suppose that a population of fish, $F(t)$ (in thousands), is given by the following model

$$\frac{dF}{dt} = 0.2\,F\left(1 - \frac{F}{100}\right) - hF,$$

where h is the intensity of fishing.

a. Assume there is no fishing ($h = 0$). Find all equilibria for this model. Sketch a graph of the right hand side of the model, then draw the phase portrait. Determine the stability of all equilibria.

b. Let $h = 0.05$, then find all equilibria for this model. Sketch a graph of the right hand side of the model, then draw the phase portrait. Determine the stability of all equilibria.

c. What level of fishing (value of h) results in the fish going extinct?

8. (Harvesting 2) Suppose that a population of fish, $F(t)$ (in thousands), is given by the following model

$$\frac{dF}{dt} = 0.2\,F\left(1 - \frac{F}{100}\right) - h,$$

where h is the annual catch of fish allowed.

a. Let $h = 1.8$, then find all equilibria for this model. Sketch a graph of the right hand side of the model, then draw the phase portrait. Determine the stability of all equilibria.

b. Let $h = 5$, then find all equilibria for this model. Sketch a graph of the right hand side of the model, then draw the phase portrait. How would the behavior change for $h > 5$?