1. a. The solution to the initial value problem,
\[
\frac{dy}{dt} = 0.3y, \quad y(0) = 20,
\]
is
\[
y(t) = 20e^{0.3t}.
\]
It requires five steps to use Euler’s method to approximate the solution using a stepsize of \( h = 0.2 \) for \( t \in [0,1] \). The Euler formula for this problem is
\[
y_{n+1} = y_n + 0.2(0.3y_n) = y_n + 0.06y_n.
\]
Below is a table showing the iterations for Euler’s solution

<table>
<thead>
<tr>
<th>( t_n )</th>
<th>( y_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>0.2</td>
<td>( y_1 = y_0 + 0.06y_0 = 20 + 0.06(20) = 21.2 )</td>
</tr>
<tr>
<td>0.4</td>
<td>( y_2 = y_1 + 0.06y_1 = 21.2 + 0.06(21.2) = 22.472 )</td>
</tr>
<tr>
<td>0.6</td>
<td>( y_3 = y_2 + 0.06y_2 = 22.472 + 0.06(22.472) = 23.8203 )</td>
</tr>
<tr>
<td>0.8</td>
<td>( y_4 = y_3 + 0.06y_3 = 23.8203 + 0.06(23.8203) = 25.2495 )</td>
</tr>
<tr>
<td>1.0</td>
<td>( y_5 = y_4 + 0.06y_4 = 25.2495 + 0.06(25.2495) = 26.7645 )</td>
</tr>
</tbody>
</table>

The exact solution for \( t = 1 \) is \( y(1) = 26.9972 \), so the percent error is \( 100 \times \frac{26.7645 - 26.9972}{26.9972} = -0.86\% \).

3. a. To verify that \( T(t) = 25 - t + 25e^{-0.2t} \) is a solution to the initial value problem
\[
T' = -k(T - (20 - t)), \quad T(0) = 50,
\]
where \( k = 0.2 \text{ hr}^{-1} \), we must verify the initial condition and that the differential equation is satisfied. First, \( T(0) = 25 - 0 + 25e^0 = 50 \), so the initial condition is satisfied.

Next we differentiate the proposed solution, giving
\[
T' = -1 + 25(-0.2)e^{-0.2t} = -1 - 5e^{-0.2t}.
\]
For the right hand side of the equation, we substitute the solution giving
\[
-k(T - (20 - t)) = -0.2 \left( 25 - t + 25e^{-0.2t} - (20 - t) \right) = -0.2 \left( 5 + 25e^{-0.2t} \right) = -1 - 5e^{-0.2t}.
\]
Thus, the equation is satisfied. The solution at \( t = 2 \) is \( T(2) = 25 - 2 + 25e^{-0.4} = 39.76^\circ \text{C} \).

b. For the Euler’s solution, we need four steps to approximate the solution using a stepsize of \( h = 0.5 \) for \( t \in [0,2] \). The Euler formula for this problem is
\[
T_{n+1} = T_n + 0.5(-k(T_n - (20 - t_n))),
\]
Below is a table showing the iterations for this Euler’s solution
The percent error is therefore 100 \\%.

For the right hand side of the equation, we substitute the solution giving
5. \( \frac{e^{-t}}{15} = 0 \). Thus, at \( t = 3 \), the approximate solution is \( R(3) = 8.99 \).

5. a. The Euler’s solution for this problem requires three steps to approximate the solution using a stepsize of \( h = 1 \) for \( t \in [0, 3] \). The Euler formula for this problem is

\[
R_{n+1} = R_n + 1.0(0.05R_n + 0.15e^{-0.02t_n}), \quad R_{n+1} = R_n - 0.05R_n + 0.15e^{-0.02t_n}.
\]

Below is a table showing the iterations for this Euler’s solution

<table>
<thead>
<tr>
<th>( t_n )</th>
<th>( R_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>( R_1 = R_0 - 0.05R_0 + 0.15e^{-0.02t_0} = 10 - 0.05(10) + 0.15e^{-0.02(0)} = 9.65 )</td>
</tr>
<tr>
<td>2</td>
<td>( R_2 = R_1 - 0.05R_1 + 0.15e^{-0.02t_1} = 9.65 - 0.05(9.65) + 0.15e^{-0.02(1)} = 9.3145 )</td>
</tr>
<tr>
<td>3</td>
<td>( R_3 = R_2 - 0.05R_2 + 0.15e^{-0.02t_2} = 9.3145 - 0.05(9.3145) + 0.15e^{-0.02(2)} = 8.9929 )</td>
</tr>
</tbody>
</table>

Thus, at \( t = 3 \), the approximate solution is \( R(3) = 8.99 \).

b. To verify that \( R(t) = 5e^{-0.05t} + 5e^{-0.02t} \) is a solution to the initial value problem

\[
R' = -0.05R + 0.15e^{-0.02t}, \quad R(0) = 10,
\]

we must verify the initial condition and that the differential equation is satisfied. First, \( R(0) = 5e^{-0.05(0)} + 5e^{-0.02(0)} = 10 \), so the initial condition is satisfied.

Next we differentiate the proposed solution, giving

\[
R' = -0.05 \left( 5e^{-0.05t} \right) - 0.02 \left( 5e^{-0.02t} \right) = -0.25e^{-0.05t} + 0.1e^{-0.02t}.
\]

For the right hand side of the equation, we substitute the solution giving

\[
-0.05R + 0.15e^{-0.02t} = -0.05 \left( 5e^{-0.05t} + 5e^{-0.02t} \right) + 0.15e^{-0.02t} = -0.25e^{-0.05t} + (-0.25 + 0.15)e^{-0.02t} = -0.25e^{-0.05t} - 0.1e^{-0.02t}.
\]

Thus, the equation is satisfied. The solution at \( t = 3 \) is \( R(3) = 5e^{-0.05*3} + 5e^{-0.02*3} = 9.012363 \). The percent error is therefore 100 \( \% \).

\[
\frac{9.012363 - 9.0124}{9.0124} = -0.22\%.
\]