1. Consider the following initial value problems:

a. $\frac{dy}{dt} = 0.3y, \quad y(0) = 20.$ b. $\frac{dy}{dt} = 10 - 0.3y, \quad y(0) = 10.$

Solve each of these initial value problems, then use Euler's method to approximate the solution using a stepsize of $h = 0.2$ for $t \in [0, 1]$. Find the approximate solution for $y(1)$, then compute the percent error between the actual solution and the approximate solution using Euler's method.

2. A population of animals that includes emigration satisfies the differential equation

$$P' = kP - m, \qquad P(0) = 100,$$

where $k = 0.1$ and $m = 2$.

a. Solve this differential equation and find $P(1)$.

b. Use Euler's method with $h = 0.2$ to approximate the solution at $t = 1$. Find the percent error between the actual solution and this approximate solution at $t = 1$.

3. The temperature of an object is initially 50°C. If it is in a room where the temperature, $T_e(t)$, is slowly decreasing with $T_e(t) = 20 - t$, then using Newton's Law of Cooling, the temperature of the object satisfies the differential equation

$$T' = -k(T - (20 - t)),$$

where $k = 0.2$ hr^{-1}.

a. Verify that the solution to this initial value problem is given by

$$T(t) = 25 - t + 25e^{-0.2t}$$

and find the temperature at $t = 2$.

b. Use Euler's method with $h = 0.5$ to approximate the solution at $t = 2$. This means that you are to the differential equation above and take four Euler's method steps. Find the percent error between the actual solution and this approximate solution at $t = 2$.

4. The body temperature of a particular animal is normally 40°C. Suppose this animal is hit by a car at midnight ($t = 0$), and the environmental temperature, $T_e(t)$, over the next few hours is slowly decreasing with $T_e(t) = 15 - t$. From Newton's Law of Cooling, the temperature of the roadkill satisfies the differential equation

$$T' = -k(T - (15 - t)),$$

where $k = 0.2$ hr^{-1}.

a. Verify that the solution to this initial value problem is given by

$$T(t) = 20 - t + 20e^{-0.2t},$$

and find the temperature of the body at 2 AM.

b. Use Euler's Method with $h = 0.5$ to approximate the temperature at $t = 2$. This means that you are to use the differential equation above and take four Euler's method steps.

5. a. Radioactive elements are often the products of the decay of another radioactive element. A differential equation describing this situation is given by the following:

$$\frac{dR}{dt} = -0.05R + 0.15e^{-0.02t}, \qquad R(0) = 10.$$

Use Euler's method with a stepsize of $h = 1$ to find the approximate solution at $t = 3$.

b. Show that the actual solution is

$$R(t) = 5e^{-0.05t} + 5e^{-0.02t}.$$

Use this solution to find the percent error of Euler's method at $t = 3$.