1. Approximate the solutions to the following equations using 3 Newton iterates starting at $x_0 = 1$. Show all 3 iterates.

a. $x^3 = 5$, b. $x^4 = 13$.

2. Consider the function

$$f(x) = 4 + 8x^2 - x^4.$$

a. Find the derivative of $f(x)$ and the second derivative, $f''(x)$.

b. Find the y-intercept. Determine any maxima or minima and all points of inflection for $f(x)$. Give both the x and y values.

c. Sketch the graph of $f(x)$. Is this function odd or even or neither?

d. One of the x-intercepts is near $x = 3$. Use Newton's Method starting with $x_0 = 3$ and performing two iterations to get a good approximation to this x-intercept.

3. Consider the function

$$f(x) = x^3 - 3x - 3.$$

a. Find all extrema and points of inflection, giving both the x and y values.

b. Sketch the graph of $f(x)$. Is this function odd or even or neither?

c. Use Newton's Method to approximate the value of the x-intercept. Start with $x_0 = 2$ and perform two iterations.

4. Let $f(x) = 4\ln(x) - x$.

a. Differentiate f and find any critical points. Determine the domain of f and sketch the graph.

b. Use Newton's Method to approximate the value of the x-intercept. Start with $x_0 = 1$ and perform two iterations to give the approximation of one zero. Then let $x_0 = 8$ and find x_1 to approximate the other zero.

5. A mass at the end of a spring oscillates about its equilibrium position ($y = 0$), executing a damped harmonic motion. Suppose that the position of the mass is given by the equation

$$y(t) = 2e^{-0.2t}\cos(2t).$$

a. Find the velocity of the spring $v(t) = y'(t)$.

b. It begins at its maximum displacement, $y(0) = 2$ cm. The minimum displacement occurs at the first time when $v(t) = 0$. Use two Newton's iterates with $t_0 = 1$ to approximate the time when this minimum displacement occurs, t_2. Find the approximate minimum displacement by evaluating $y(t_2)$.