1. By standard differentiation of trig functions, 
   \[ f'(x) = 0 - 4(3 \cos(3x)) = -12 \cos(3x) \].

3. This uses the chain rule. Let \( u(x) = \cos(2x) \), then we can write 
   \[ f(x) = 3u^3(x) \], which by the chain rule gives 
   \[ f'(x) = 9u^2(x) \frac{du}{dx} \]. But \( \frac{du}{dx} = -2 \sin(2x) \), so it follows that 
   \[ f'(x) = -18 \cos^2(2x) \sin(2x) \].

5. This uses the product rule, so 
   \[ f'(x) = e^{2x}(-4 \sin(4x)) + \cos(4x)(2e^{2x}) = 2e^{2x} (\cos(4x) - 2 \sin(4x)) \].

7. This is another chain rule. If \( u(x) = x^2 - \cos(2x^3) \), then \( f(x) = u^4(x) \), and \( f'(x) = 4u^3(x) \frac{du}{dx} \). But from the chain rule again, \( \frac{du}{dx} = 2x - (-\sin(2x^3)) \cdot 6x^2 \). Combining these results gives 
   \[ f'(x) = 4 \left( x^2 - \cos(2x^3) \right)^3 \left( 2x + 6x^2 \sin(2x^3) \right) \].

9. Begin by writing this expression as 
   \[ f(x) = \sin(x^2)^{-4} \], then the chain rule with \( u(x) = \sin(x^2) \) gives 
   \[ f'(x) = -4u^{-5}(x) \frac{du}{dx} \]. Since \( \frac{du}{dx} = 2x \cos(x^2) \), it follows that 
   \[ f'(x) = \frac{-8x \cos(x^2)}{\sin^5(x^2)} \].

11. Differentiation of \( y \) uses the product rule, so 
   \[ y' = e^x(-\sin(x)) + \cos(x) \cdot e^x = e^x (\cos(x) - \sin(x)) \].
   The relative extrema occur when \( y' = 0 \), which only occurs when \( \cos(x) - \sin(x) = 0 \) or \( \cos(x) = \sin(x) \). For \( x \in [0, 2\pi] \), the sine and cosine functions are equal at \( \frac{\pi}{4} \) and \( \frac{5\pi}{4} \). It follows that there is relative maximum at \( \left( \frac{\pi}{4}, e^{\pi/4}/\sqrt{2} \right) \simeq (0.7854, 1.551) \) and a minimum at \( \left( \frac{5\pi}{4}, -e^{5\pi/4}/\sqrt{2} \right) \simeq (3.927, -35.89) \). There is an absolute maximum at the endpoint \( x = 2\pi \) with \( y = e^{2\pi} \simeq 535.5 \). Below is the graph.
13. a. The mass follows \( y(t) = 2 \cos(10t) \). Since the cosine function is bounded between \(-1\) and \(1\), it follows that the maximum displacements occur with \( y(t) = 2 \) cm at times when \( 10t = 2n\pi \) (for any integer \( n = 0, 1, 2, \ldots \)) or \( t = \frac{n\pi}{5} \). The minimum displacements occur with \( y(t) = -2 \) cm at times when \( 10t = (2n + 1)\pi \), where \( n = 0, 1, 2, \ldots \), or \( t = \frac{\pi}{10} + \frac{n\pi}{5} \). The period satisfies \( 10T = 2\pi \) or \( T = \frac{\pi}{5} \simeq 0.6283 \) sec.

b. The velocity is \( v(t) = y'(t) = -20 \sin(10t) \), and the acceleration is \( a(t) = v'(t) = -200 \cos(10t) \). The maximum velocity is \( 20 \) cm/sec occurring when \( 10t = \frac{3\pi}{2} + 2n\pi \), where \( n = 0, 1, 2, \ldots \), which is equivalent to \( t = \frac{3\pi}{20} + \frac{n\pi}{5} \) sec.

15. a. The vertical force can be written \( F(t) = A(\sin(6t))(1 - a \sin(18t)) \). Since we are given that \( 0 < a < 1 \), it follows that \( 1 - a \sin(18t) > 0 \). (The sine function is at most one, so we are subtracting a quantity less than one from one.) \( F(t) = 0 \) when \( 6t = n\pi \), where \( n = 0, 1, 2, \ldots \). Thus, \( t = \frac{n\pi}{6} \) sec, so the foot is on the ground for \( \frac{n\pi}{6} \) sec.

b. The derivative of \( F(t) \) uses the product rule, so
\[
F'(t) = A ((\sin(bt))(-3ab \cos(3bt)) + (1 - a \sin(3bt))(b \cos(bt)))
\]

At \( t = \frac{\pi}{12} \), we have \( \cos(6 \cdot \frac{\pi}{12}) = \cos(\frac{\pi}{2}) = 0 \) and \( \cos(18 \cdot \frac{\pi}{12}) = \cos(\frac{3\pi}{2}) = 0 \). With these values substituted into the derivative, \( F'(\frac{\pi}{12}) = 0 \). The maximum value is \( F(\frac{\pi}{12}) = A \sin(6 \cdot \frac{\pi}{12})(1 - a \sin(18 \cdot \frac{\pi}{12})) = A(1 + a) \), since \( \sin(6 \cdot \frac{\pi}{12}) = \sin(\frac{\pi}{2}) = 1 \) and \( \sin(18 \cdot \frac{\pi}{12}) = \sin(\frac{3\pi}{2}) = -1 \).

17. a. The function \( S(\theta) \) can be written
\[
S(\theta) = \frac{4V\sqrt{3}}{3R} + \frac{3R^2}{2} \left( \sqrt{3} \sin^{-1}(\theta) - \frac{\cos(\theta)}{\sin(\theta)} \right).
\]
The first term is a constant, the second uses the chain rule, and the last uses the quotient rule. It follows that
\[
\frac{dS}{d\theta} = \frac{3R^2}{2} \left( -\sqrt{3} \sin^{-2}(\theta) \cos(\theta) - \frac{\sin(\theta)(-\sin(\theta)) - \cos(\theta) \cos(\theta)}{\sin^2(\theta)} \right)
\]
\[
= \frac{3R^2}{2} \left( 1 - \sqrt{3} \cos(\theta) \right) \frac{1}{\sin^2(\theta)},
\]
since \( \cos^2(\theta) + \sin^2(\theta) = 1 \).

b. The derivative is zero when \( 1 - \sqrt{3} \cos(\theta) = 0 \) or \( \cos(\theta) = \frac{1}{\sqrt{3}} \simeq 0.5774 \). This minimizes the surface area. It follows that \( \arccos \left( \frac{1}{\sqrt{3}} \right) \simeq 0.9553 \simeq 54.7^\circ \).