Differentiate the following functions:

1. $f(x) = 5 - 4\sin(3x)$

2. $f(x) = 2\cos(7x) - x^2$

3. $f(x) = 3\cos^3(2x)$

4. $f(x) = \ln(2 + \sin(3x))$

5. $f(x) = e^{2x}\cos(4x)$

6. $f(x) = x\sin(x^2 - \pi)$

7. $f(x) = \left(x^2 - \cos(2x^3)\right)^4$

8. $f(x) = \cos(4x)\sin^3(4x)$

9. $f(x) = \dfrac{1}{\sin^4(x^2)}$

10. $f(x) = \dfrac{\cos(3x)}{2 - \sin(3x)}$

11. Sketch a graph of the following function:

$$y = e^x\cos(x).$$

Find all extrema for $x \in [0, 2\pi]$.

12. Sketch a graph of the following function:

$$y = \frac{1}{\sin(2x)}.$$

Find all extrema and asymptotes for $x \in [0, 2\pi]$. (Note: This is the graph for $y = \csc(2x)$.)

13. A mass at the end of a spring without any damping executes simple harmonic motion. This motion is described by the equation

$$y(t) = A\cos(\omega t),$$

where y is the position of the spring from rest and t is the time. Suppose that $A = 2$ cm, $\omega = 10$, and time is in seconds.

a. Find the maxmimum and minimum displacements (positions) of the mass. What is the period of the oscillation for this mass?

b. Find expressions for the velocity ($v(t) = y'(t)$) and the acceleration ($a(t) = y"(t)$) of the mass. Also, determine the maximum velocity of the mass and when it occurs.

14. During normal breathing, the volume of the lungs varies between 2200 ml and 2800 ml with about one breath every 3 seconds. This exchange of air is called the *tidal volume*.

a. Assume that the volume of air in the lungs satisfies a cosine function written in the following manner:

$$V(t) = A + B\cos(\omega t),$$

where A, B, and ω are constants and t is in seconds. Use the data above and techniques from the previous section to create a model, *i.e.*, find A, B, and ω that simulates the normal breathing of an individual.

b. Differentiate the function above to find the rate of exchange of air in ml/sec as a function of time, t. Find the maximum rate of air that is exhaled (loss of volume) and when this occurs during the first 3 seconds.

c. Sketch a graph of both the volume of air in the lungs and its derivative.

15. When a person is walking, the magnitude of the vertical force acting on one foot can be approximated by the function

$$F(t) = A(\sin(bt)\,(1 - a\sin(3bt))\,,$$

where t is the time in seconds, A is proportional to the weight of the person, and b and a are constants with $b > 0$ and $0 < a < 1$.

a. Suppose that $b = 6$, then find the times when the force is zero. The difference between these times is the length of time that the foot is on the ground. How long is the foot on the ground?

b. Find the derivative of $F(t)$. Then show that $F(t)$ has a maximum at $\pi/12$ when $b = 6$. What is this maximum value of $F(t)$?

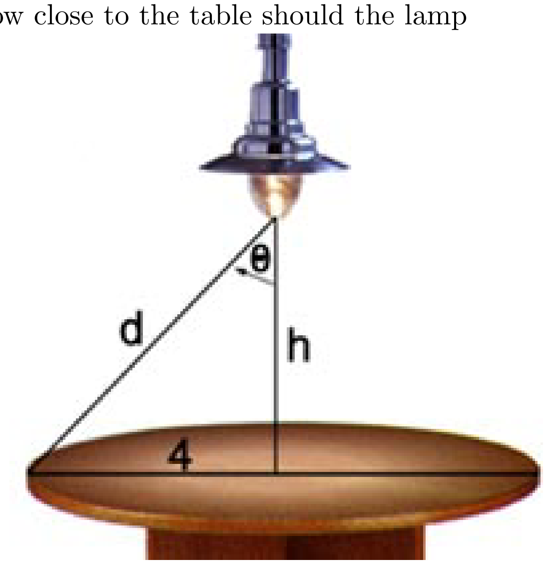
16. A lamp with an adjustable height hangs above a circular table with a radius of 4 ft. A right triangle is formed between the center of the table, the lamp, and the edge of the table. The height h is the edge of the triangle between the center and the lamp, and the hypotenuse, d, is the distance from the lamp to the edge. Suppose that the illumination of the edge of the table is directly proportional to the cosine of the angle θ, which is formed between the edge h and the hypotenuse d of the triangle, and is inversely proportional to the square of the distance d between the lamp and the edge of the table.

a. If I is the illumination at the edge of the table, then show that

$$I(\theta) = \frac{k}{16}\cos(\theta)\sin^2(\theta).$$

Find the derivative, $I'(\theta)$.

b. Show that $I'(\theta) = 0$ when $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \sqrt{2}$. How close to the table should the lamp be to maximize the illumination at the edge of the table?



17. In a beehive, each cell is a regular hexagonal prism (with radius and side length of R), open at one end with a trihedral angle at the other end. The trihedral end consists of 3 identical rhombuses that meet at a vertex with the face of these rhombuses forming an angle of θ with the vertical of the cell. Measurements of these honeycomb cells have indicated that bees minimize the surface area of these cells for a fixed volume, V (to hold the pupae). From the geometry of the cells, it can be shown that the total surface area S is given by the formula:

$$S(\theta) = \frac{4V\sqrt{3}}{3R} - \frac{3R^2}{2}\frac{\cos(\theta)}{\sin(\theta)} + \frac{3R^2\sqrt{3}}{2}\frac{1}{\sin(\theta)}.$$

a. Calculate $dS/d\theta$. Use the trig identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to simplify your expression.

b. Find the $\cos(\theta)$ that minimizes the surface area and show that it's about 54.7°.