1. a. In the lecture notes, we examined the ODE

$$y'' - y = 0.$$

Show that $y_1(t) = e^t$ and $y_2(t) = e^{-t}$ are solutions to the differential equation. In addition, show that this pair form a linearly independent set using the definition of linear independence.

b. Also, show that $y_1(t) = \sinh(t)$ and $y_2(t) = \sinh(1-t)$ are solutions to the differential equation. In addition, show that this pair form another linearly independent set.

2. Consider the following second order linear homogeneous differential equation:

$$y'' - 2ay' + (a^2 + b^2)y = 0,$$

where the parameters are fixed and positive, so assume $a > 0$ and $b > 0$.

a. Find the general solution to this ordinary differential equation (ODE).

b. Find the unique solution to the initial value problem (IVP), where the initial conditions for the ODE are:

$$y(0) = y_0 \qquad \text{and} \qquad y'(0) = z_0.$$

c. Now consider the ODE with boundary conditions:

$$y(0) = A \qquad \text{and} \qquad y(x_0) = B.$$

Give conditions on the boundary condition parameters, A, B, and $x_0 > 0$, such that this boundary value problem (BVP) has:
(i) A unique solution. (ii) No solution. (iii) Infinitely many solutions.
When this BVP has a unique solution, give the solution to the BVP.