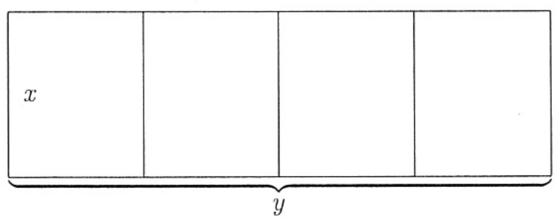
March 24, 2017 Math 124b Name Key

1. (4 pts) Differentiate the following: (**DON'T simplify**):

$$f(x) = \left(\cos(4x+2) - e^{-2x^4}\right)^5.$$

$f'(x) = 5\left(\cos(4x+2) - e^{-2x^4}\right)^4\left(-4\sin(4x+2) + 8x^3e^{-2x^4}\right)$

2. (7 pts) You are an impoverished graduate student in ecology and must design an experimental plot for study with 4 identical sections, which are fenced in according to the design below. Define x to be the width of the plot and y be the length of the plot (total length of fencing on the top and bottom of the diagram below). Suppose that you have 140 m of fencing available. Find the dimensions of the plot that maximizes the area of each plot in your study area. Be sure to show the formulae for area and length of fence in the space below for partial credit.



Objective Function $A(x,y) = xy/4$

Constraint Condition $P(x,y) = 5x + 2y = 140$

$y = 70 - \frac{5}{2}x \qquad A(x) = \frac{x}{4}\left(70 - \frac{5}{2}x\right) = \frac{35}{2}x - \frac{5}{8}x^2$

$A'(x) = \frac{35}{2} - \frac{5}{4}x = 0 \quad \Rightarrow x_c = 14$

$x =$ 14 m and $y =$ 35 m

Area of each plot = 122.5 m^2

$8\left(1+\frac{x}{4}-x+5\right)=8\left(6-\frac{3x}{4}\right)$

$=2(24-3x)$

3. (9 pts) Draw the graph of the following function:

$$y=\frac{8(x-5)}{(1+\frac{x}{4})^4}.$$

Find the y and x-intercepts. Find any asymptotes (vertical and/or horizontal). Give the derivative of the function, then determine extrema, (local maxima and/or minima, including the x and y values). Sketch this function, $y(x)$. **(If a maximum or minimum or particular asymptote doesn't exist, then write "NONE.")**

y-intercept = -40 $\quad$ x-intercept(s) = 5

Horizontal Asymptote y = 0 $\quad$ Vertical Asymptote x = -4

$$y'(x)=8\,\frac{(1+\frac{x}{4})^4-(x-5)\,4(1+\frac{x}{4})^3\cdot\frac{1}{4}}{(1+\frac{x}{4})^8}=\frac{6(8-x)}{(1+\frac{x}{4})^5}$$

x_{max} = 8 $\quad$ $y(x_{max})$ = 0.2963 $\quad$ x_{min} = None $\quad$ $y(x_{min})$ = None

Sketch of GRAPH