March 10, 2017 Math 124b Name Key

1. (7 pts) Many ecological studies require that the subject studied is correlated with the temperature of the environment (especially insects and plants). Over a 20 hour period, data are collected on the temperature, $T(t)$in degrees Celsius. The temperature data are found to best fit the cubic polynomial

$$T(t) = 0.01(1600 - 135t + 27t^2 - t^3),$$

where t is in hours (valid for $0 \leq t \leq 20$).

a. Find the rate of change in temperature per hour, $\frac{dT}{dt}$. What is the rate of change in the temperature at 3 AM, $t = 3$? Also, compute $T''(t)$. When is the rate of change in the temperature increasing the most and what is that maximum rate of increase?

$T'(t) = -0.03(t^2 - 18t + 45) = -0.03(t-3)(t-15)$

$T'(t) =$ $-0.01(3t^2 - 54t + 135)$ $T'(5) =$ 0.60

$T''(t) =$ $-0.03(2t - 18)$

Rate of maximum increase at $t_{inc} =$ 9 $T'(t_{inc}) =$ 1.08 °C/hr

b. Use the derivative to find when the minimum and maximum temperatures occur. Give the temperatures at those times.

$t_{max} =$ 15 $T(t_{max}) =$ 22.75 °C

$t_{min} =$ 3 $T(t_{min}) =$ 14.11 °C

2. (3 pts) Differentiate the following: (**DON'T simplify**):

$$g(x) = 12\cos(5(x-2)) - 15e^{-x/5} + \frac{6}{\sqrt{x^3}}.$$

$g(x) = 12\cos(5(x-2)) - 15e^{-x/5} + 6x^{-3/2}$

$g'(x) =$ $-60\sin(5(x-2)) + 3e^{-x/5} - 9x^{-5/2}$

3. (10 pts) Suppose that a natural hormone in the body responds to a drug by increasing shortly after the drug is administered. Suppose that the concentration of hormone, $H(t)$, is given by the function,

$$H(t) = 11 + 16\left(e^{-0.05t} - e^{-0.67t}\right),$$

where t is in hours. Find the derivative $H'(t)$. Find when the hormone achieves its maximum concentration and determine what its maximum concentration is. Sketch a graph of H showing the H-intercept, the maximum, and any horizontal asymptotes.

H-intercept = 11 Horizontal Asymptote H = 11

$H'(t) = 16(-0.05e^{-0.05t} + 0.67e^{-0.67t})$

$H'(t) = 0 \Rightarrow$

$0.05e^{-0.05t} = 0.67e^{-0.67t}$

$e^{0.62t} = \frac{0.68}{0.05}$

$t = \frac{\ln(68/5)}{0.62}$

t_{max} = 4.1859 $H(t_{max})$ = 23.010

Sketch of GRAPH