

1. (7 pts) Many ecological studies require that the subject studied is correlated with the temperature of the environment (especially insects and plants). Over a 20 hour period, data are collected on the temperature,  $T(t)$  in degrees Celsius. The temperature data are found to best fit the cubic polynomial

$$T(t) = 0.01(1600 - 135t + 27t^2 - t^3),$$

where  $t$  is in hours (valid for  $0 \leq t \leq 20$ ).

a. Find the rate of change in temperature per hour,  $\frac{dT}{dt}$ . What is the rate of change in the temperature at 3 AM,  $t = 3$ ? Also, compute  $T''(t)$ . When is the rate of change in the temperature increasing the most and what is that maximum rate of increase?

$$T'(t) = -0.03(t^2 - 18t + 45) = -0.03(t - 3)(t - 15)$$

$$T'(t) = \underline{-0.01(3t^2 - 54t + 135)} \quad T'(5) = \underline{0.60}$$

$$T''(t) = \underline{-0.03(2t - 18)}$$

$$\text{Rate of maximum increase at } t_{inc} = \underline{9} \quad T'(t_{inc}) = \underline{1.08} \text{ } ^\circ\text{C/hr}$$

b. Use the derivative to find when the minimum and maximum temperatures occur. Give the temperatures at those times.

$$t_{max} = \underline{15} \quad T(t_{max}) = \underline{22.75} \text{ } ^\circ\text{C}$$

$$t_{min} = \underline{3} \quad T(t_{min}) = \underline{14.11} \text{ } ^\circ\text{C}$$

2. (3 pts) Differentiate the following: (DON'T simplify):

$$g(x) = 12 \cos(5(x-2)) - 15e^{-x/5} + \frac{6}{\sqrt{x^3}}$$

$$g(x) = 12 \cos(5(x-2)) - 15e^{-x/5} + 6x^{-3/2}$$

$$g'(x) = \underline{-60 \sin(5(x-2)) + 3e^{-x/5} - 9x^{-5/2}}$$

3. (10 pts) Suppose that a natural hormone in the body responds to a drug by increasing shortly after the drug is administered. Suppose that the concentration of hormone,  $H(t)$ , is given by the function,

$$H(t) = 11 + 16(e^{-0.05t} - e^{-0.67t}),$$

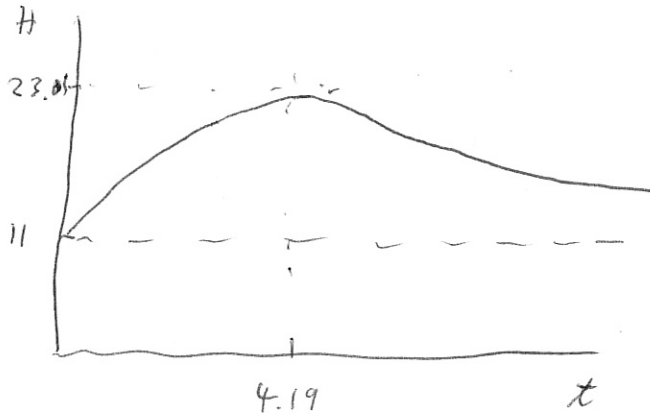
where  $t$  is in hours. Find the derivative  $H'(t)$ . Find when the hormone achieves its maximum concentration and determine what its maximum concentration is. Sketch a graph of  $H$  showing the  $H$ -intercept, the maximum, and any horizontal asymptotes.

$H$ -intercept = 11      Horizontal Asymptote  $H =$  11

$$H'(t) = \underline{16(-0.05e^{-0.05t} + 0.67e^{-0.67t})}$$

$t_{max} =$  4.1859       $H(t_{max}) =$  23.010

Sketch of GRAPH



$$\begin{aligned}
 H'(t) = 0 &\Rightarrow \\
 0.05e^{-0.05t} &= 0.67e^{-0.67t} \\
 e^{0.62t} &= \frac{0.67}{0.05} \\
 t &= \frac{\ln(67/5)}{0.62}
 \end{aligned}$$