Give all answers to at least **4 significant figures**.

1. In 1946, A. C. Crombie studied a number of populations of insects with the amount of food supplied strictly regulated. One study examined *Rhizopertha dominica*, the lesser grain borer. A slightly modified set of his population data is given in the table below:

Week	Population	Week	Population
0	2	16	205
2	2	18	261
4	3	20	302
6	17	22	330
8	65	24	315
10	119	26	333
12	130	28	350
14	175	30	332

In this problem, we want to compare the discrete logistic growth and Beverton-Holt models. For an adult population, P_n, the discrete logistic growth model is given by:

$$P_{n+1} = f(P_n) = rP_n - mP_n^2,$$

where the constants r and m are determined from the data. Similarly, the Beverton-Holt model is given by:

$$P_{n+1} = B(P_n) = \frac{aP_n}{1 + bP_n},$$

where the constants a and b are determined from the data. This problem is similar to the beetle problem in Lab 1, where you find the updating function, then use the updating function to simulate the time series.

a. Begin this problem by finding the **two updating functions**, $f(P_n)$ and $B(P_n)$, given above. Plot P_{n+1} vs. P_n, which you can do by entering the adult population data from times 0-28 for P_n and times 2-30 for P_{n+1}. (Be sure that P_n is on the horizontal axis.) For the logistic growth function, $f(P_n)$, use Excel's trendline with its polynomial fit of order 2 and with the intercept set to 0 (under options). Write the best constants, r and m, and determine the sum of square errors between the data and the updating function.

You continue by finding the best fitting Beverton-Holt updating function, $B(P_n)$. For this function, we find the least squares best fit of this function to the data. The initial guess of a should be equal to the value of r found above, while the initial guess for b is r/M, where M is the highest population in the data. From the sum of square errors, use Excel's Solver to find the best fitting parameters a and b and give the sum of square errors. Compare this number to the one for the logistic growth updating function. Which updating function fits the data better.

b. Find the equilibria for both the logistic growth and Beverton-Holt models using the best fitting parameters found above. Write the derivative of both updating functions. Evaluate the derivatives at each of the equilibria, then discuss the behavior (stability) of these models near their equilibria.

c. Take each of these models with an initial population that is to be determined by the sum of square errors between the time series data and the simulated models. (Two simulations, one for the logistic growth model and one for the Beverton-Holt model.) Use Solver to find the value of the best initial conditions for each of the models. (Modify only the initial condition for each of the models, using the parameters found above for the updating functions in the model.) List the simulated values at times 10, 20, and 30 weeks for each of the models. Write the sum of square errors between the models with the best initial population. Which model fits the data better? Is there a significant difference between these two models. Discuss how well your simulation matches the data in the table.

2. Pollution in fresh water is a major ecological problem for today's society. One of the central laws responsible for cleaning the water in the U. S. is the Clean Water Act of 1972, which was enacted in no small part due to the horrible condition of Lake Erie in the middle of the last century.

a. We begin this problem using a simple model for pollution in Lake Erie based on data from Rainey [2]. A basic mathematical model for the concentration of a pollutant, $c(t)$, in a well-mixed lake is given by the differential equation:

$$\frac{dc}{dt} = \frac{kr}{V} - \frac{r}{V}c, \qquad c(0) = c_0,$$

where k is the concentration of pollutant entering the lake at a rate r (which is also the rate water leaves), V is the volume of the lake, and c_0 is the initial concentration of the pollutant in the lake. The volume of Lake Erie is $V = 460$ km^3, and its flow rate is $r = 175$ km^3/yr. Solve this differential equation, including the unknown constants k and c_0.

b. Assume that a relatively new pesticide is detected in the lake, and is monitored over a period of time. Below is a table of the concentrations of the pesticide found in Lake Erie.

t (yr)	0	1	2	3	4	5
c (ppb)	2.1	2.8	3.5	3.9	4.3	4.5

Use the solution from Part a and Excel's Solver to find the best fitting constants k and c_0 (least squares best fit) for this model based on the data in the table above. Also, give the sum of square errors for these best constants.

c. If the pesticide entering the lake is stopped completely, $k = 0$, then determine how long until the concentration of the pollutant drops to half the amount seen in Year 5 from the table. Find the concentrations 5 and 10 years after the pesticide is stopped in Year 5.

d. Unfortunately, pesticides remain in the environment and are slowly washed out. Assume that a law is enacted to ban this particular pesticide in Year 5. If it is found that the residual pesticide enters in an exponentially declining manner, then a new model for the concentration of pollutant in Lake Erie is

$$\frac{dc}{dt} = \frac{4.5r\,e^{-0.15t}}{V} - \frac{r}{V}c, \qquad c(0) = c_0,$$

where the constants r and V are from Part a and $c_0 = 4.5$. Use Euler's method with a stepsize of $h = 0.25\tilde{(}\text{yr})$ to estimate the solution of this differential equation on the time interval $t \in [0, 10]$. Give the concentrations at times $t = 1, 2, 3, 5, 7,$ and 10 years after the pesticide is stopped. Compare the answers at $t = 5$ and 10 to the ones in Part c.

e. Use Maple's **dsolve** to find the actual solution to the differential equation above. Determine how long it takes for the level of the pollutant to drop to half the amount found in Year 5. Use the solution to determine the percent error between the Euler solution in Part d and the actual solution.

[1] A. C. Crombie (1946) On competition between different species of graminivorous insects, *Proc. R. Soc. (B)*, **132**, 362-395.

[2] R. H. Rainey (1967), Natural displacement of pollution from the Great Lakes, *Science*, **155**, 1242-1243.