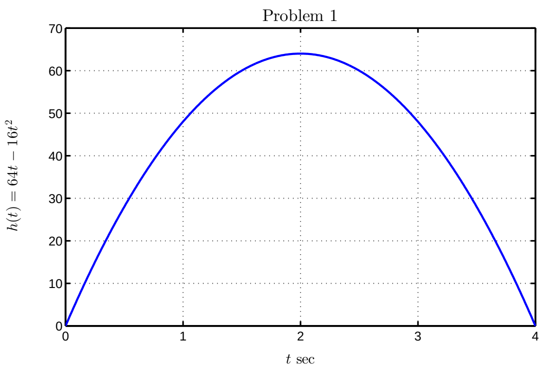
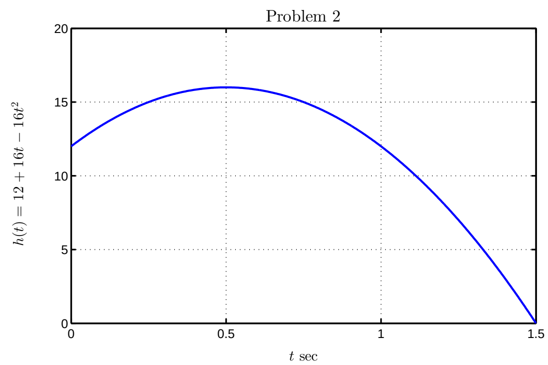
1. a. The function $h(t) = 64t - 16t^2 = -16t(t-4)$ is a parabola. The t-intercept with $t > 0$ is when the ball hits the ground, so $t = 4$ sec. The maximum height occurs at $t = 2$ (the vertex) with $h(2) = 64$ ft. The graph of $h(t)$ is below on the left.

b. The average velocity for $t \in [1,2]$ is $\frac{h(2)-h(1)}{2-1} = \frac{64-48}{2-1} = 16$ ft/sec, for $t \in [1, 1.1]$ is $\frac{51.04-48}{1.1-1} = 30.4$ ft/sec, and for $t \in [1, 1.001]$ is $\frac{48.03198-48}{1.001-1} = 31.98$ ft/sec.





2. a. The position of the cat is $h(t) = 12 + 16t - 16t^2$. The maximum height occurs between the two t-intercepts. We solve

$$h(t) = -4(4t^2 - 4t - 3) = -4(2t+1)(2t-3) = 0 \qquad \text{or} \qquad t = -0.5 \quad \text{and} \quad t = 1.5.$$

Halfway between the intercepts is $t = 0.5$, so the maximum is $h(0.5) = 12 + 16(0.5) - 16(0.5)^2 = 16$ ft. Since the cat achieves its maximum height at $t = 0.5$ sec with a maximum height of 16 ft, the cat cannot reach the bird (at 18 ft).

b. Average velocity is shown below

$$\begin{aligned} v_{ave} &= \frac{h(0.5) - h(0)}{0.5 - 0} = \frac{16 - 12}{0.5} = 8 \text{ ft/sec} \\ v_{ave} &= \frac{h(1) - h(0.5)}{1 - 0.5} = \frac{12 - 16}{0.5} = -8 \text{ ft/sec} \end{aligned}$$

Thus, the average velocity of the cat for $t \in [0, \frac{1}{2}]$ is 8 ft/sec, while $t \in [\frac{1}{2}, 1]$ is -8 ft/sec.

c. The cat hits the ground when $h(t) = 0$ at time $t = 1.5$ sec. The velocity of impact is found by considering a time $t = 1.5 + \tau$. The height is $h(1.5 + \tau) = 12 + 16(1.5 + \tau) - 16(1.5 + \tau)^2 = 0 - 32\tau - 16\tau^2$. It follows that the velocity at 1.5 is approximated by

$$v_{ave} = \frac{h(1.5 + \tau) - h(1.5)}{\tau} = \frac{-32\tau - 16\tau^2 - 0}{\tau} = -32 - 16\tau.$$

For small τ, this quantity converges to $v(1.5) = -32$ ft/sec. Thus, the cat hits the ground at $t = 1.5$ sec with a velocity of $v(1.5) = -32$ ft/sec. A sketch of the graph for the height of the cat as a function of t is shown above on the right.

3. a. Since the kangaroo can jump 240 cm, this is the highest value of $h(t)$, which is the vertex of the quadratic function $h(t)$. First we find the t-intercepts for $h(t)$, which satisfy

$$h(t) = v_0 t - 490t^2 = t(v_0 - 490t) = 0.$$

It follows that either $t = 0$ or $t = \frac{v_0}{490}$. The t-value of the vertex is the midpoint, so $t = \frac{v_0}{980}$. It follows that the height of the vertex satisfies

$$h\left(\frac{v_0}{980}\right) = v_0\left(\frac{v_0}{980}\right) - 490\left(\frac{v_0}{980}\right)^2 = \frac{v_0^2}{1960} = 240.$$

So the initial velocity is given by

$$v_0 = \sqrt{1960(240)} = 280\sqrt{6} \approx 685.86 \text{ cm/sec.}$$

The kangaroo is in the air until

$$t = \frac{v_0}{490} = \frac{4}{7}\sqrt{6} \approx 1.3997 \text{ sec.}$$

b. The average velocity of the kangaroo between $t = 0$ and $t = 1$ is

$$v_{ave} = \frac{h(1) - h(0)}{1 - 0} = 280\sqrt{6} - 490 \approx 195.86 \text{ cm/sec.}$$

The average velocity of the kangaroo between $t = .99$ and $t = 1$ is

$$v_{ave} = \frac{h(1) - h(.99)}{1 - 0.99} = \frac{280\sqrt{6}(1 - 0.99) - 490(1 - .99^2)}{1 - .99} \approx -289.24 \text{ cm/sec.}$$

4. a. With the function $f(x) = 1 - x^2$, we are given the point (1, 0) on the curve. We use the point-slope form of the line to find each line in the desired sequence. First, we evaluate the function at each of the x-values listed, so

$$\begin{aligned} f(2) &= 1 - 2^2 = -3 \\ f(1.5) &= 1 - 1.5^2 = -1.25 \\ f(1.1) &= 1 - 1.1^2 = -0.21 \\ f(1.01) &= 1 - 1.012 = -0.0201. \end{aligned}$$

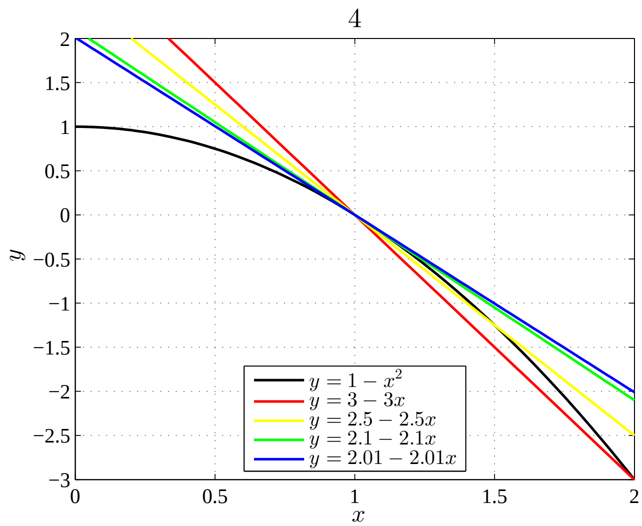
The slopes of the secant lines satisfy:

$$\begin{aligned} m &= \frac{f(2) - f(1)}{2 - 1} = \frac{-3 - 0}{1} = -3 \\ m &= \frac{f(1.5) - f(1)}{1.5 - 1} = \frac{-1.25 - 0}{0.5} = -2.5 \\ m &= \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{-0.21 - 0}{0.1} = -2.1 \\ m &= \frac{f(1.01) - f(1)}{1.01 - 1} = \frac{-0.0201 - 0}{0.01} = -2.01. \end{aligned}$$

To find the y-intercept b of the secant lines, we solve $0 = m(1) + b$, so $b = -m$. It follows that the sequence of secant lines are:

$$\begin{aligned} y &= -3x + 3 \\ y &= -2.5x + 2.5 \\ y &= -2.1x + 2.1 \\ y &= -2.01x + 2.01. \end{aligned}$$

b. It is easy to see that the slopes of the secant lines are converging to $m = -2$, so the tangent line at $x = 1$ is $y = -2x + 2$. The slope at $x = 1$ is -2, so the derivative of $f(x)$ at $x = 1$ is -2. Below is a graph of the secant lines.



5. a. With $f(x) = 3x - x^2$, the slope of the secant line satisfies:

$$\begin{aligned} m &= \frac{f(1+h) - f(1)}{h} \\ &= \frac{(3(1+h) - (1+h)^2) - (3(1) - 1^2)}{(1+h) - 1} \\ &= \frac{h - h^2}{h} = 1 - h. \end{aligned}$$

b. When h gets small, $m = 1$. It follows that the equation of the tangent line is

$$y = x + 1.$$

6. a. With $f(x) = 3x - 4$, the slope of the secant line satisfies:

$$m = \frac{f(1+h) - f(1)}{h}$$

$$
\begin{aligned}
&= \frac{(3(1+h)-4)-(3(1)-4)}{(1+h)-1} \\
&= \frac{3h}{h} = 3.
\end{aligned}
$$

b. We see that the slope of the secant line is $m = 3$ independent of h. It follows that the tangent line is the line $f(x)$ or $y = 3x - 4$.

7. a. With $f(x) = \frac{4}{x+5}$, the slope of the secant line satisfies:

$$
\begin{aligned}
m &= \frac{f(-3+h)-f(-3)}{h} \\
&= \frac{\frac{4}{(-3+h)+5} - \frac{4}{-3+5}}{(-3+h)-(-3)} = \frac{\frac{4}{(2+h)} - \frac{4}{2}}{h} \\
&= \frac{1}{h}\left(\frac{2(4)-(2+h)(4)}{2(2+h)}\right) \\
&= \frac{-2}{2+h}.
\end{aligned}
$$

b. We see that when h gets small, the slope approaches $m = -1$. It follows that the equation of the tangent line is

$$y - 2 = -(x+3) \qquad \text{or} \qquad y = -x - 1.$$