1. a. A ball thrown vertically with an initial velocity of 64 ft/sec upwards satisfies the equation

$$h(t) = 64t - 16t^2,$$

with $h(t)$ being the height of the ball in feet.

a. Find the time when the ball reaches its maximum height, t_{max}, and what that maximum height, h_{max}, is. At what time does the ball hit the ground?

b. The average velocity is the difference in heights divided by the time between the measurements. Find the average velocity between $t = 1$ and $t = 2$, between $t = 1$ and $t = 1.1$, and between $t = 1$ and $t = 1.001$.

2. a. A cat is sitting on a ledge 12 ft above the ground. A bird flies by at a height of 18 ft above the ground. The cat leaps up with a vertical velocity of 16 ft/sec trying to catch the bird. If we ignore air resistance and use an acceleration from gravity of $g = -32$ ft/sec^2, then the height of the cat above the ground, $h(t)$, is given by the formula

$$h(t) = 12 + 16t - 16t^2.$$

Find the maximum height that the cat achieves and how long it takes to reach that maximum height. Can the cat catch the bird?

b. Find the average velocity of the cat for the intervals $t \in [0, \frac{1}{2}]$ and $t \in [\frac{1}{2}, 1]$.

c. Determine the time when the cat hits the ground and the velocity of impact. You should sketch a graph of the height of the cat as a function of t.

3. A kangaroo can leap vertically 240 cm. The initial velocity, v_0 is unknown, so we want to determine it from the data on how high it can jump using Newton's law of gravity. The equation describing the height of the kangaroo is

$$h(t) = v_0 t - 490t^2.$$

a. Use the information above to determine the animal's initial upward velocity, v_0, then find how long the kangaroo is in the air (the hang time).

b. Find the average velocity of the kangaroo between $t = 0$ and $t = 1$ and between $t = 0.99$ and $t = 1$.

4. Consider the function $f(x) = 1 - x^2$.

To find the equation of the tangent line at the point $x = 1$ or $(1, 0)$, we find a sequence of secant lines passing through $(1, 0)$.

a. Let one point on all secant lines be $(1, 0)$. Find the secant lines where the second point has $x = 2$ and $y = f(2)$, $x = 1.5$ and $y = f(1.5)$, $x = 1.1$ and $y = f(1.1)$, and $x = 1.01$ and $y = f(1.01)$. You should sketch a graph of $f(x)$ and the secant lines.

b. Use these secant lines to predict the equation of the tangent line. The slope of the tangent line gives the derivative at $x = 1$, so find the derivative of $f(x)$ at $x = 1$.

5. Consider the function:
$$f(x) = 3x - x^2.$$

a. Find the slope of the secant line through the points $(1, f(1))$ and $(1 + h, f(1 + h))$. (Note that your answer should include an h.)

b. Let h get small and determine the slope of the tangent line through $(1, f(1))$, which gives the value of the derivative of $f(x)$ at $x = 1$.

6. Consider the function:
$$f(x) = 3x - 4.$$

a. Find the slope of the secant line through the points $(1, f(1))$ and $(1 + h, f(1 + h))$. (Note that your answer should include an h.)

b. Let h get small and determine the slope of the tangent line through $(1, f(1))$, which gives the value of the derivative of $f(x)$ at $x = 1$.

7. Consider the function:
$$f(x) = \frac{4}{x + 5}.$$

a. Find the slope of the secant line through the points $(-3, f(-3))$ and $(-3+h, f(-3+h))$. (Note that your answer should include an h.)

b. Let h get small and determine the slope of the tangent line through $(-3, f(-3))$, which gives the value of the derivative of $f(x)$ at $x = -3$.