1. Radian measures are computed as degrees using the constant multiplier

$$\frac{180^\circ}{\pi}$$

It follows that:

radian (x)	degree (θ)	radian (x)	degree (θ)	radian (x)	degree (θ)
$\frac{5\pi}{6}$	150°	$\frac{2\pi}{4}$	90°	$\frac{\pi}{3}$	60°
$\frac{5\pi}{2}$	450°	4π	720°		

2. Degree measures are computed as radians using the constant multiplier

$$\frac{\pi}{180^\circ}$$

It follows that:

degree (θ)	radian (x)	degree (θ)	radian (x)	degree (θ)	radian (x)
-320°	$-\frac{16\pi}{9}$	260°	$\frac{13\pi}{9}$	-290°	$-\frac{29\pi}{18}$
190°	$\frac{19\pi}{18}$	-120°	$-\frac{2\pi}{3}$		

3 and 4. Sine and cosine values are computed from tables or a calculator, set for radian measure. Some standard measures should be known exactly.

radian (x)	$\sin(x)$	$\cos(x)$	radian (x)	$\sin(x)$	$\cos(x)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\pi}{2}$	1	0
π	0	-1	2π	0	1

5. Since $\cos(\theta) = \frac{3}{5}$, which is adjacent over hypotenuse, by Pythagorean's Theorem, the opposite side is $\sqrt{5^2 - 3^2} = 4$. It follows that $\sin(\theta) = \frac{4}{5}$. Alternately, we can use trig identities as follows: $\sin^2(\theta) + \cos^2(\theta) = 1$ so $\sin(\theta) = \sqrt{1 - \cos^2(\theta)} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$.

For questions 6- 12 compare the equation with the definitions from the lecture

$$y(t) = A + B\cos(\omega(t - \psi))$$

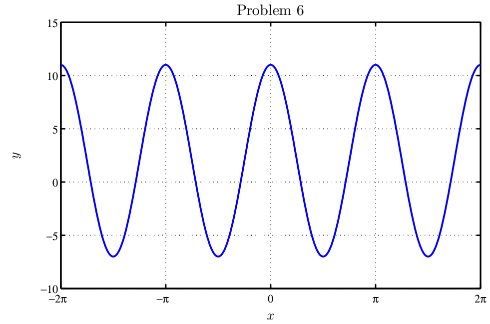
and

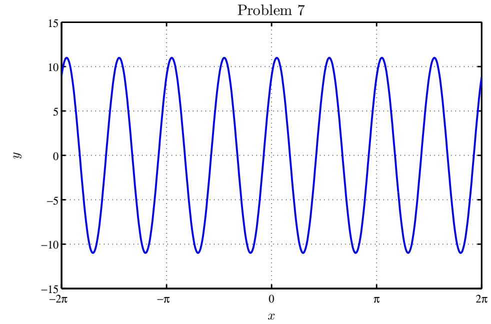
$$y(t) = A + B\sin(\omega(t - \psi))$$

The vertical shift parameter is A.
The amplitude parameter is B.
The frequency parameter is ω.
The phase shift parameter is ψ.
The period is denoted by $T = \frac{2\pi}{\omega}$.

6. For the function, $y(x) = 2 + 9\cos(2x)$ we have: Amplitude $= 9$. Period $= \pi$. Vertical shift $= 2$.

Evaluation $y(\pi/2) = 2 + 9\cos(\frac{2\pi}{2}) = -7$. A graph of this function appears below on the left.





.

7. For the function, $y(x) = -11\cos\left(4\left(x + \frac{\pi}{5}\right)\right)$, we have: Amplitude $= 11$. Period $= \frac{\pi}{2}$. Phase shift $= -\frac{\pi}{5}$.

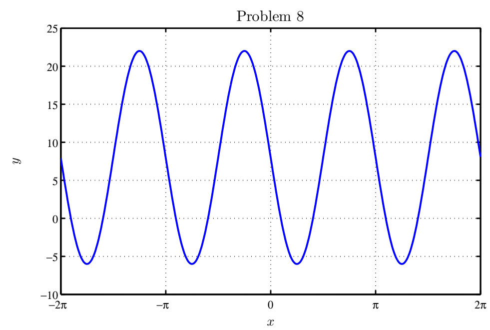
Evaluation $y(\pi/2) = -11\cos\left(4\left(\frac{\pi}{2} + \frac{\pi}{5}\right)\right) = 8.8992$. A graph of this function appears above on the right.

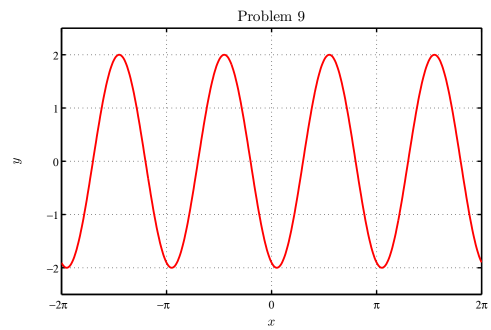
8. a. For the function, $y(x) = 8 - 14\sin(2x)$, we have: Amplitude $= 14$. Period $= \pi$. Vertical shift $= 8$.

Evaluate $y(\pi/2) = 8 - 14\sin\left(2(\frac{\pi}{2})\right) = 8$.

b. The amplitude, frequency, and vertical shift remain the same, so we want an equivalent model $y(x) = 8 + 14\sin(2(x - \phi))$ for some ϕ. By changing the sign of the amplitude to positive, we shift by half a period or $\phi = \frac{\pi}{2}$.

A graph of this function appears below on the left.





.

9. a. For the function, $y(x) = -2\sin(2x + \frac{2\pi}{5})$, we have: Amplitude $= 2$. Period $= \pi$. Phase shift $= -\frac{\pi}{5}$.

Evaluation $y(\pi/2) = -2\sin\left(2(\frac{\pi}{2}) + \frac{2\pi}{5}\right) = 1.9021$.

A graph of this function appears above on the right.

b. The amplitude and frequency stay the same, so we want an equivalent model $y(x) = 2\sin(2(x-\phi))$ for some ϕ. To change the sign of the amplitude, we shift by half a period or solve

$$2\left(x+\frac{\pi}{5}\right)-\pi = 2(x-\phi) \qquad \text{or} \qquad \phi = -\frac{\pi}{5}+\frac{\pi}{2} = \frac{3\pi}{10}$$

Thus, the equivalent model is

$$y(x) = 2\sin\left(2\left(x-\frac{3\pi}{10}\right)\right).$$

10. a. For the function, $y(x) = 8\cos(3(x+\frac{\pi}{4}))-5$, we have: Amplitude $= 8$. Period $= \frac{2\pi}{3}$. Vertical shift $= -5$. Phase shift $= -\frac{\pi}{4}$.

Evaluation $y(\pi/2) = 8\cos\left(3\left(\frac{\pi}{2}+\frac{\pi}{4}\right)\right)-5 = 0.6569$.

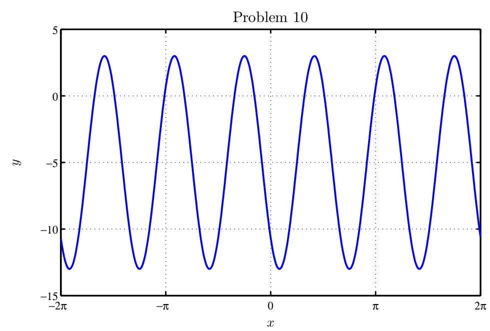
A graph of this function appears below on the left.

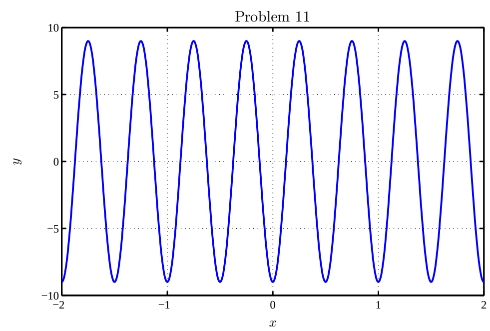
b. The amplitude, frequency, and vertical shift remain the same, so we want an equivalent model $y(x) = 8\cos(3(x-\phi))-5$ for some $\phi \in [0, \frac{2\pi}{3})$. An equivalent model is obtained by shifting the phase shift by one period or

$$\phi = -\frac{\pi}{4}+\frac{2\pi}{3} = \frac{5\pi}{12}.$$

Thus, the equivalent model is

$$y(x) = 8\cos\left(3\left(x-\frac{5\pi}{12}\right)\right)-5.$$





11. a. For the function, $y(x) = -9\cos(4\pi(x+4))$, we have: Amplitude $= 9$. Period $= \frac{1}{2}$. Phase shift $= -4$.

Evaluation $y(4) = -9\cos(8\pi(4+4)) = -9$.

A graph of this function appears above on the right.

b. The amplitude and frequency stay the same, so we want an equivalent model $y(x) = 9\cos(4\pi(x-\phi))$ for some $\phi \in [0, \frac{1}{2})$. To change the sign of the amplitude, we shift by half a period or solve

$$4\pi(x+4)-\pi = 4\pi(x-\phi_1) \qquad \text{or} \qquad \phi_1 = \frac{1}{4}-4.$$

However, ϕ_1 is not in $[0, \frac{1}{2})$. Thus, we take ϕ to be $\phi_1 + nT$, an integer multiple of the period $T = \frac{1}{2}$. Specifically, take $n = 8$, then $\phi = \phi_1 + 8(\frac{1}{2}) = \frac{1}{4}$, so the equivalent model is

$$y(x) = 9\cos\left(4\pi\left(x - \frac{1}{4}\right)\right).$$

12. a. For the function, $y(x) = 3\sin\left(2\left(x + \frac{\pi}{4}\right)\right) + 7$, we have: Amplitude = 3. Period = π. Vertical shift = 7. Phase shift = $-\frac{\pi}{4}$.

Evaluation $y(\frac{\pi}{2}) = 3\sin\left(2\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right) + 7 = 4$.

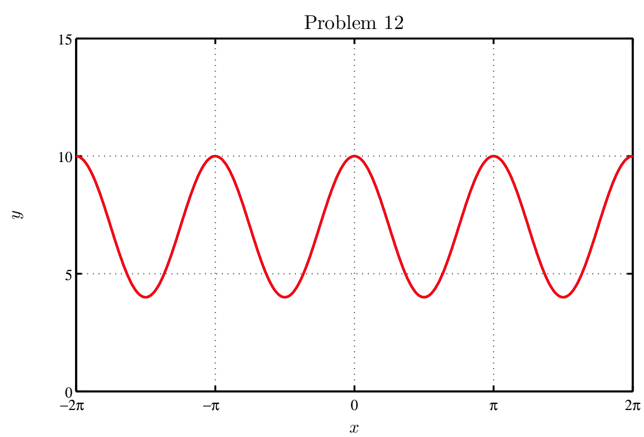
A graph of this function appears below.

b. The amplitude, vertical shift, and frequency stay the same, so we want an equivalent model $y(x) = 3\sin(2(x - \phi)) + 7$ for some $\phi \in [0, \pi)$. Since the phase shift in the original problem is negative, we simply have to add an integral multiple of the period. Thus, we take:

$$\phi = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}.$$

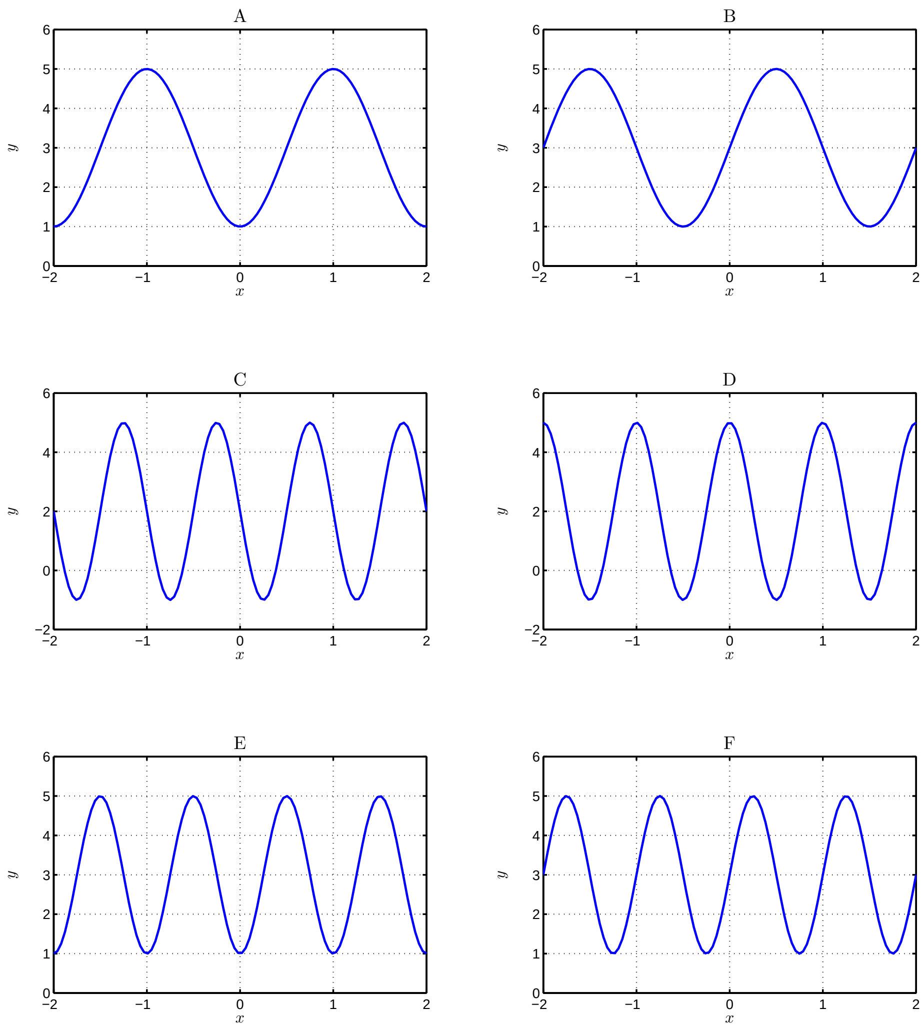
The equivalent model is

$$y(x) = 3\sin\left(2\left(x - \frac{3\pi}{4}\right)\right) + 7.$$



13. The equation with the letter for its graph are below.

$$
\begin{array}{lllll}
1. & y & = & 3 - 2\cos(2\pi(x+1)) & \text{Graph E} \\
2. & y & = & 3 - 2\cos(\pi x) & \text{Graph A} \\
3. & y & = & 2 + 3\cos(2\pi(x-1)) & \text{Graph D} \\
4. & y & = & 2 - 3\sin(2\pi x) & \text{Graph C} \\
5. & y & = & 3 + 2\sin(2\pi(x+1)) & \text{Graph F} \\
6. & y & = & 3 + 2\sin(\pi x) & \text{Graph B}
\end{array}
$$



.

14. The angle of depression is 50° or $\frac{5\pi}{18}$ radians. Thus, the angle between the vertical line (hawk and ground) and line to rabbit is $\frac{\pi}{2} - \frac{5\pi}{18} = \frac{2\pi}{9} = \alpha$. The distance to the rabbit is the hypotenuse of this right-angled triangle.

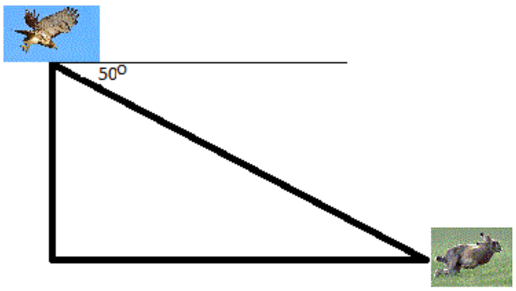
$$\cos(\alpha) = \frac{320}{h}$$

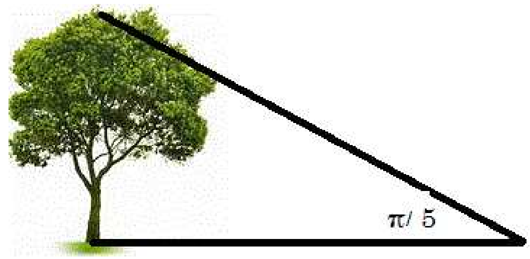
$$h = \frac{320}{\cos(\alpha)} = 417.73 \text{ ft.}$$

The distance along the ground between the hawk and the rabbit is d

$$\tan(\alpha) = \frac{d}{320}$$
$$d = 320 \cdot \tan(\alpha) = 268.51 \text{ ft.}$$

The diagram is below on the left.





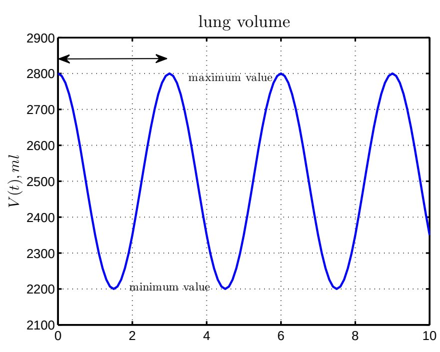
15. The diagram is above on the right. The height of the tree, h, satisfies:

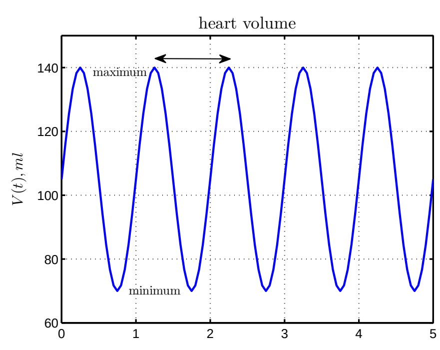
$$\tan\left(\frac{\pi}{5}\right) = \frac{h}{35}$$
$$h = 35 \cdot \tan\left(\frac{\pi}{5}\right) = 25.43 \text{ m.}$$

16. We want a model of the form $V(t) = A + B\cos(\omega t)$, where the maximum is 2800 ml and the minimum is 2200 ml. Since A is the average volume, $A = \frac{2200+2800}{2} = 2500$. The amplitude, B, is the distance from the average to the maximum, so $B = 2800 - 2500 = 300$. We are given that the period of one breath is $\frac{1}{24}$ min, so the frequency $\omega = \frac{2\pi}{1/24} = 48\pi \simeq 150.8$. It follows that the model satisfies:

$$V(t) = 2500 + 300\cos(48\pi t).$$

Clearly, the maximum is 2800 ml at $t = 0$ and every $\frac{1}{24}$ min or 2.5 sec interval. The minimum is 2200 ml at $t = \frac{1}{48}$ min or 1.25 sec and every 2.5 sec interval. The graph over 10 sec is shown below on the left.





17. a. We want a model of the form $B(t) = a + b\sin(\omega t)$, where the maximum in the heart is 140 ml of blood and the minimum is 70 ml of blood. Its mean volume is given by

$$a = \frac{140 + 70}{2} = 105 \text{ ml}.$$

The value of the variation from the mean is $b = 140 - 105 = 35$ ml. There are 60 pulses/min, so the period $T = \frac{1}{60}$. Thus, we find $\omega = \frac{2\pi}{T} = 120\pi$. It follows that

$$B(t) = 105 + 35\sin(120\pi t),$$

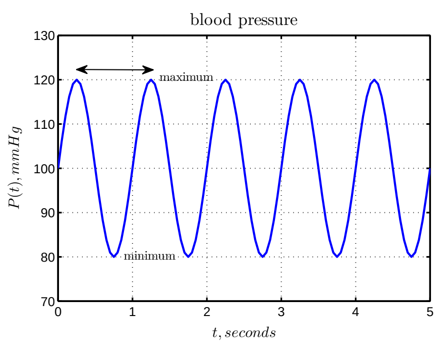
which is graphed above right. After 1 second $B(\frac{1}{60}) = 105 + 35\sin(\frac{120\pi}{60}) = 105$.

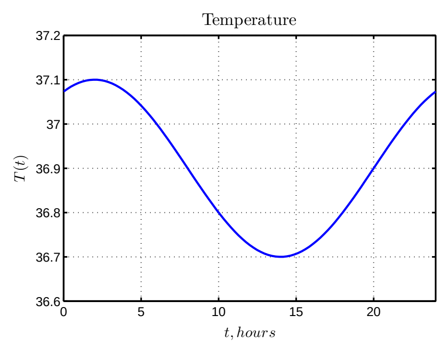
b. We want a similar model of the form $P(t) = c + d\sin(\omega t)$, where the maximum pressure is 120 mm Hg and the minimum is 80 mm Hg. We find the mean pressure $c = \frac{120+80}{2} =$ 100 mm Hg. The value of the variation from the mean is $d = 20$ mm Hg. There are 60 pulses/min, so the period $T = \frac{1}{60}$ min, as in Part a. Thus, we find $\omega = \frac{2\pi}{T} = 120\pi$, and we write the specific equation

$$P(t) = 100 + 20\sin(120\pi t),$$

which is graphed below on the left.

c. For an equivalent model of the form $P(t) = C + D\cos(\nu(t - \phi))$, the vertical shift, amplitude, and frequency stay the same as Part b. Thus, $C = 100$, $D = 20$, and $\nu = 120\pi$. From the lecture notes, we see that the phase shift for going from a sine model to a cosine model is a quarter period to the left, so $\phi = \frac{1}{240}$. Alternately, the maximum of the sine model above occurs at $t_{max} = \frac{1}{240}$, and the maximum of the cosine model matches its phase shift, so again $\phi = \frac{1}{240}$.





18. a. We want a model of the form $T(t) = A + B\cos(\omega(t\phi))$, where the maximum body temperature is 37.1 °C (at $t = 2$) and the minimum body temperature is 36.7 °C (at $t = 14$). The mean temperature gives $A = \frac{37.1+36.7}{2} = 36.9$. The amplitude satisfies $B = 37.1 - 36.9 = 0.2$. Since the period is 24 hr, the frequency is $\omega = \frac{2\pi}{24} = \frac{\pi}{12} \simeq 0.2618$. For the cosine model, the phase shift occurs at the maximum, so $\phi = 2$. The best model is given by:

$$T(t) = 36.9 + 0.2\cos\left(\frac{\pi}{12}(t - 2)\right).$$

A graph for the temperature of this individual for one day is shown above on the right. The body temperature at 9 AM satisfies:

$$T(9) = 36.9 + 0.2\cos\left(\frac{\pi}{12}(9-2)\right) = 36.85°\text{C}.$$

b. An equivalent temperature model of the form

$$T(t) = C + D\sin(\nu(t-\psi)),$$

has the same vertical shift, amplitude, and frequency, so $C = 36.9$, $D = 0.2$, and $\nu = \frac{\pi}{12}$. The phase shift, ψ, for the sine function is a quarter period from the cosine function. This shift can also be found by matching the maximum, so

$$\frac{\pi}{2} = \frac{\pi}{12}(2-\psi) \qquad \text{or} \qquad \psi = -4,$$

which gives the phase shift $\psi_2 \in [-24, 0)$. To obtain $\psi \in [0, 24)$ we add one period, so $\psi = 20$. This gives the model:

$$T(t) = 36.9 + 0.2\sin\left(\frac{\pi}{12}(t-20)\right).$$

19. a. We want a model of the form $T(t) = A + B\sin(\omega(t\phi))$, where the minimum body temperature is 75 °F (at $t = 3$) and the maximum body temperature is 104 °F (at $t = 15$). The mean temperature gives $A = \frac{75+104}{2} = 89.5$. The amplitude satisfies $B = 104 - 89.5 = 14.5$. Since the period is 24 hr, the frequency is $\omega = \frac{2\pi}{24} = \frac{\pi}{12} \simeq 0.2618$. For the sine model, the phase shift occurs at the maximum, so

$$\frac{\pi}{2} = \frac{\pi}{12}(15-\phi) \qquad \text{or} \qquad \phi = 9.$$

The best model is given by:

$$T(t) = 89.5 + 14.5\sin\left(\frac{\pi}{12}(t-9)\right).$$

A graph for the temperature of the iguana for one day is shown below. The body temperature at 6 AM satisfies:

$$T(6) = 89.5 + 14.5\sin\left(\frac{\pi}{12}(6-9)\right) = 79.25°\text{F}.$$

b. Since the average of the model is 89°F, which suggests the iguana body temperature is over 88°F for over 12 hr, making this a reasonable estimate. Alternately, we can solve when $T(t) = 88°$F. We can solve $T(t) = 89.5 + 14.5\sin\left(\frac{\pi}{12}(t-9)\right) = 88$, and find that the time at which the temperature first rises above 88° F is 8.604 hours, and it cools below 88° F at 21.396 hours, so the time above 88° F is $21.396 - 8.604 = 12.79$ hours or 12 hours 48 min.

c. An equivalent temperature model of the form

$$T(t) = C + D\cos(\nu(t-\psi)),$$

has the same vertical shift, amplitude, and frequency, so $C = 89.5$, $D = 14.5$, and $\nu = \frac{\pi}{12}$. The phase shift, ψ, for the cosine function is a quarter period from the sine function. This shift occurs at the maximum, so $\psi = 15$. This gives the model:

$$T(t) = 89.5 + 14.5\cos\left(\frac{\pi}{12}(t - 15)\right).$$

To find a phase shift, $\psi_2 \in [-24, 0)$, we subtract a period, so $\psi_2 = 15 - 24 = -9$.