1. The equation $\frac{dy}{dt} = 3t^2y$ is a separable differential equation, so

$$\int \frac{dy}{y} = \int 3t^2 dt \qquad \text{or} \qquad \ln|y| = t^3 + C.$$

Solving for y gives

$$y(t) = e^{t^3+C} = Ae^{t^3}, \qquad \text{with} \qquad A = e^C.$$

The initial condition is $y(0) = 44 = Ae^0 = A$, so the solution is

$$y(t) = 44e^{t^3}.$$

2. The equation $\frac{dy}{dt} = 3 - 2y = -2\left(y - \frac{3}{2}\right)$ is a linear differential equation. We make the substitution $z(t) = y(t) - \frac{3}{2}$ with $z(0) = y(0) - \frac{3}{2} = 3.5$. The translated problem is

$$\frac{dz}{dt} = -2z, \qquad \text{with} \qquad z(0) = 3.5,$$

so the solution is

$$z(t) = 3.5e^{-2t} = y(t) - \frac{3}{2}.$$

It follows that the solution is

$$y(t) = 1.5 + 3.5e^{-2t}.$$

3. The equation $\frac{dy}{dt} = (t+2)e^{-y}$ is a separable differential equation, so

$$\int e^y dy = \int (t+2) dt \qquad \text{or} \qquad e^y = \frac{t^2}{2} + 2t + C.$$

Taking logarithms, we solve for $y(t)$,

$$y(t) = \ln\left|\frac{t^2}{2} + 2t + C\right|,$$

which with the initial condition $y(0) = 19$ gives $\ln|C| = 19$ or $C = e^{19}$. Thus, the solution is

$$y(t) = \ln\left|\frac{t^2}{2} + 2t + e^{19}\right|.$$

4. The equation $\frac{dy}{dt} = 3t^2y^2$ is a separable differential equation, so

$$\int y^{-2} dy = 3\int t^2 dt \qquad \text{or} \qquad \frac{y^{-1}}{-1} = t^3 + C.$$

This is solved for $y(t)$ giving

$$y(t) = -\frac{1}{t^3 + C}.$$

With the initial condition, $y(0) = 5 = -\frac{1}{C}$ or $C = -0.2$. It follows that

$$y(t) = -\frac{1}{t^3 - 0.2}.$$

5. The equation $\frac{dy}{dt} = \frac{(1-2t)}{2y}$ is a separable differential equation, so

$$2\int y dy = \int (1-2t)dt \qquad \text{or} \qquad y^2 = t - t^2 + C.$$

This is solved for $y(t)$ giving

$$y(t) = \pm\sqrt{t - t^2 + C}.$$

With the initial condition, $y(0) = 4 = \sqrt{C}$ or $C = 16$. It follows that

$$y(t) = \sqrt{t - t^2 + 16}.$$

6. The equation $\frac{dy}{dt} = 3t^2 + 12$ is an integrable differential equation, so

$$y(t) = \int (3t^2 + 12)dt = t^3 + 12t + C.$$

With the initial condition, $y(0) = 8 = C$. It follows that

$$y(t) = t^3 + 12t + 8.$$

7. The equation $\frac{dy}{dt} = 2ty$ is a separable differential equation, so

$$\int \frac{dy}{y} = 2\int t dt \qquad \text{or} \qquad \ln|y| = t^2 + C.$$

This is solved for $y(t)$ giving

$$y(t) = e^{t^2 + C} = Ae^{t^2}.$$

With the initial condition, $y(0) = 5 = A$. It follows that

$$y(t) = 5e^{t^2}.$$

8. The equation $(1 + 2y)\frac{dy}{dt} = 2t$ is a separable differential equation, so

$$\int (1 + 2y)dy = 2\int t dt \qquad \text{or} \qquad y + y^2 = t^2 + C.$$

One completes the square to give

$$\left(y(t) + \frac{1}{2}\right)^2 = t^2 + C + \frac{1}{4} \qquad \text{or} \qquad y(t) = -\frac{1}{2} \pm \sqrt{t^2 + C + \frac{1}{4}}.$$

With the initial condition, $y(2) = 0 = -\frac{1}{2} + \sqrt{C + \frac{17}{4}}$ or $C = -4$. It follows that

$$y(t) = -\frac{1}{2} + \sqrt{t^2 - \frac{15}{4}}.$$

9. The equation $t\frac{dy}{dt} = 2y$ is a separable differential equation, so

$$\int \frac{dy}{y} = 2\int \frac{dt}{t} \qquad \text{or} \qquad \ln|y| = 2\ln|t| + C.$$

Solving for $y(t)$, we have

$$y(t) = e^{2\ln|t|+C} = e^{\ln(t)^2}e^C = At^2.$$

With the initial condition, $y(1) = 4 = A$. It follows that

$$y(t) = 4t^2.$$

10. The equation $\frac{dy}{dt} = 2\cos(2t)y^2$ is a separable differential equation, so

$$\int y^{-2}dy = 2\int (\cos(2t))\,dt \qquad \text{or} \qquad -\frac{1}{y(t)} = \sin(2t) + C.$$

Solving for $y(t)$, we have

$$y(t) = -\frac{1}{\sin(2t) + C}.$$

With the initial condition, $y(0) = 1 = -\frac{1}{C}$ or $C = -1$. It follows that

$$y(t) = \frac{1}{1 - \sin(2t)}.$$

11. a. The equation $\frac{dV}{dt} = -kV^{2/3}$ is a separable differential equation, so

$$\int V^{-2/3}dV = -k\int dt \qquad \text{or} \qquad 3V^{1/3} = -kt + C.$$

Solving for $V(t)$ gives

$$V(t) = \left(\frac{C - kt}{3}\right)^3,$$

where the initial condition $V(0) = 8$ implies that $C = 6$. The other condition is $V(3) = 1$, so

$$V(3) = 1 = (2 - k)^3 \qquad \text{or} \qquad k = 1.$$

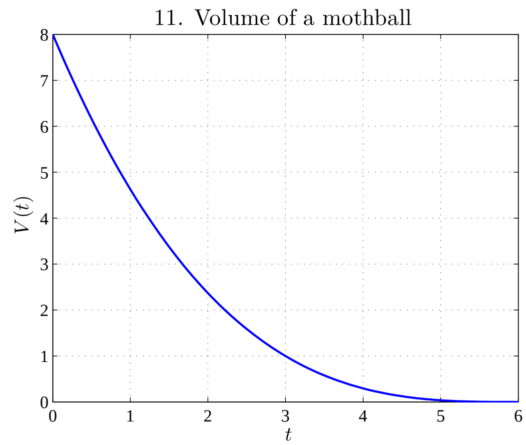
It follows that the solution is:

$$V(t) = \left(2 - \frac{t}{3}\right)^3.$$

b. When the mothball disappears, $V(t) = 0$, so

$$V(t) = \left(2 - \frac{t}{3}\right)^3 = 0 \qquad \text{or} \qquad t = 6.$$

The graph of $V(t)$ vs. t is shown below.



12. a. The equation $\frac{dV}{dt} = 0.04V^{3/4}$ is a separable differential equation, so

$$\int V^{-3/4}dV = 0.04 \qquad \text{or} \qquad 4V^{1/4} = 0.04t + C.$$

Solving for $V(t)$ gives

$$V(t) = \left(0.01t + \frac{C}{4}\right)^4,$$

where the initial condition $V(0) = 1$ implies that $C = 4$. It follows that the solution is:

$$V(t) = (0.01t + 1)^4.$$

b. For the cell to double its volume, we solve

$$V(t) = (0.01t + 1)^4 = 2 \qquad \text{or} \qquad (0.01t + 1) = \sqrt[4]{2}.$$

It follows that $t = 100(\sqrt[4]{2} - 1) = 18.92$ min is the time for the cell to double its volume.

13. a. The solution to the Malthusian growth equation is

$$Y(t) = 2000e^{0.08t}.$$

It doubles when

$$4000 = 2000e^{0.08t} \qquad \text{or} \qquad 0.08t = \ln(2).$$

It follows that $t = 12.5\ln(2) = 8.66$ hr.

b. The DE $\frac{dY}{dt} = (0.08 - 0.002t)Y$ is a separable equation, so

$$\int \frac{dY}{Y} = \int (0.08 - 0.002t)dt \qquad \text{or} \qquad \ln|Y(t)| = 0.08t - 0.001t^2 + C.$$

Thus,

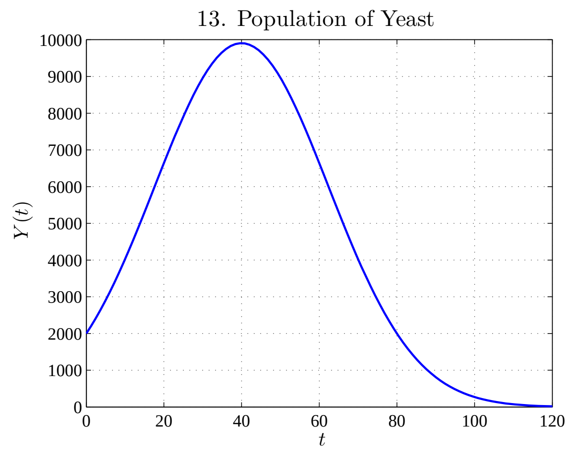
$$Y(t) = e^{0.08t - 0.001t^2 + C} = Ae^{0.08t - 0.001t^2}.$$

With the initial condition, $Y(0) = 2000 = A$, so

$$Y(t) = 2000e^{0.08t - 0.001t^2}.$$

c. The maximum occurs when $\frac{dY}{dt} = 0$, which is true when $0.08 - 0.002t = 0$ or $t = 40$. From the solution, $Y(40) = 2000e^{0.08(40) - 0.001(40)^2} = 2000e^{1.6} \approx 9906$.

The population is 2000 when the exponent of the solution is zero, so $0.08t - 0.001t^2 = 0$, when $t = 0$ or 80 hrs. Thus, the population returns to 2000 after 80 hrs. The graph is shown below.



14. a. The DE $\frac{dP}{dt} = (b - at)P$ is a separable equation, so

$$\int \frac{dP}{P} = \int (b - at)dt \qquad \text{or} \qquad \ln|P| = bt - \frac{at^2}{2} + C.$$

Thus,

$$P(t) = e^{bt - \frac{at^2}{2} + C} = Ae^{bt - \frac{1}{2}at^2}.$$

With the initial condition, $P(0) = 6.94 = A$, so

$$P(t) = 6.94e^{bt - \frac{1}{2}at^2}.$$

From the data, $P(20) = 7.47$ and $P(40) = 7.72$.

$$7.47 = 6.94e^{20b - 200a} \qquad \text{and} \qquad 7.72 = 6.94e^{40b - 800a}.$$

Taking logarithms,

$$\ln\left(\frac{7.47}{6.94}\right) = 20b - 200a \qquad \text{and} \qquad \ln\left(\frac{7.72}{6.94}\right) = 40b - 800a.$$

Taking the 2^{nd} equation times 0.5 and adding to the first gives

$$\begin{aligned} 200a &= \ln\left(\frac{7.47}{6.94}\right) - 0.5\ln\left(\frac{7.72}{6.94}\right) \\ a &= 0.005\ln\left(\frac{7.47}{\sqrt{7.72 \times 6.94}}\right) \approx 0.000101685. \end{aligned}$$

It follows that $b = 0.0046965$.

b. The year 2000 is $t = 50$. $P(50) = 6.94e^{0.0046965(50)-0.000050843(50)^2} \approx 7.7293$ million. The actual census data is 8.13 million, so the percent error is $100\frac{(7.73-8.13)}{8.13}\% = -4.93\%$ from the actual census data.

c. The maximum population occurs when $\frac{dP}{dt} = (b - at)P = 0$, so $t = \frac{b}{a} \approx 46.19$ yr. This gives

$$P_{max} = 6.94e^{0.004697(46.19)-0.00005084(46.19)^2} \approx 7.735.$$

Therefore, according to the model, the maximum population occurs in about 1996, and is about 7.735 million. This is clearly not correct based on the census data in 2000.

15. For convenience, let 1941 correspond to t = 0 and define P(t) to be the population of India. The Malthusian growth model is

$$\frac{dP}{dt} = rP, \qquad P(0) = 319 \text{ million}.$$

The solution to this is $P(t) = 319e^{rt}$. Since the population in 1961 ($t = 20$) is 438 million, the Malthusian growth model gives

$$P(20) = 319e^{20r} = 438, \qquad \text{so} \qquad r = \frac{1}{20}\ln\left(\frac{438}{319}\right) \approx 0.015851.$$

The model produces $P(10)) = 319e^{0.015851(10)} = 373.79$ million. The percent error is $100\frac{|373.79-361|}{361}\% = 3.54\%$.

b. The DE $\frac{dP}{dt} = (at + b)P$ is a separable equation, so

$$\int \frac{dP}{P} = \int (at + b)dt \qquad \text{or} \qquad \ln|P| = \frac{at^2}{2} + bt + C.$$

Thus,

$$P(t) = e^{\frac{at^2}{2}+bt+C} = Ae^{\frac{1}{2}at^2+bt}.$$

With the initial condition, $P(0) = 319 = A$, so

$$P(t) = 319e^{\frac{1}{2}at^2+bt}.$$

From the data, $P(10) = 361$ and $P(20) = 438$.

$$361 = 319e^{50a+10b} \qquad \text{and} \qquad 438 = 319e^{200a+20b}.$$

Taking logarithms,

$$\ln\left(\frac{361}{319}\right) = 50a + 10b \qquad \text{and} \qquad \ln\left(\frac{438}{319}\right) = 200a + 20b.$$

Taking the 2^{nd} equation times 0.5 and subtracting to the first gives

$$\begin{aligned} 50a &= 0.5\ln\left(\frac{438}{319}\right) - \ln\left(\frac{361}{319}\right) \\ a &= 0.02\ln\left(\frac{\sqrt{438 \times 319}}{361}\right) \approx 0.00069654. \end{aligned}$$

It follows that $b = 0.00888598$. Thus, the nonautonomous Malthusian growth model becomes

$$P(t) = 319e^{0.00034827t^2 + 0.00888598t}.$$

c. The Malthusian growth model gives the population in 1991 as $P(50) = 708.7$ million, while the nonautonomous Malthusian growth model gives $P(50) = 1,215.6$ million. The percent error from the actual population of 846 million for the Malthusian growth model is 16.2%, while the percent error for the nonautonomous Malthusian growth model is 43.7%. So in this case, the Malthusian growth model is the better model.