1. Consider $f(x) = x^4 + 7x^3 - 2x^2 - 4x + 3$. The derivative is a straight application of the power rule.

$$f'(x) = 4x^3 + (7 \cdot 3)x^2 - (2 \cdot 2)x - 4 = 4x^3 + 21x^2 - 4x - 4.$$

2. Begin by writing $h(t)$ in powers of t.

$$\begin{aligned} h(t) &= t^3 - 5t + \frac{1}{2} - \frac{1}{t^2} \;=\; t^3 - 5t + \frac{1}{2} - t^{-2} \\ h'(t) &= 3t^2 - 5 - (-2)t^{-3} = 3t^2 - 5 + \frac{2}{t^3}. \end{aligned}$$

3. Begin by writing $p(z)$ in powers of z.

$$\begin{aligned} p(z) &= z^{\frac{1}{3}} + 4.7z^2 - 7\sqrt{z^5} \;=\; z^{\frac{1}{3}} + 4.7z^2 - 7z^{\frac{5}{2}} \\ p'(z) &= \frac{1}{3}z^{-\frac{2}{3}} + 2 \cdot 4.7z - \frac{5}{2} \cdot 7z^{\frac{3}{2}} \\ &= \frac{1}{3}z^{-\frac{2}{3}} + 9.4z - \frac{35\sqrt{z^3}}{2}. \end{aligned}$$

4. Begin by writing $q(w)$ in powers of w.

$$\begin{aligned} q(w) &= 3w^{-0.4} + 2.1w^5 - \frac{2}{\sqrt{w}} \;=\; 3w^{-0.4} + 2.1w^5 - 2w^{-\frac{1}{2}} \\ q'(w) &= -0.4 \cdot 3w^{-1.4} + 5 \cdot 2.1w^4 - \frac{-1}{2} \cdot 2w^{-\frac{3}{2}} \\ &= -1.2w^{-1.4} + 10.5w^4 + \frac{1}{\sqrt{w^3}}. \end{aligned}$$

5. Begin by writing $g(x)$ in powers of x.

$$\begin{aligned} g(x) &= A - \frac{B}{x^3} + \frac{C}{\sqrt{x}} - Dx^4 \;=\; A - Bx^{-3} + Cx^{-\frac{1}{2}} - Dx^4 \\ g'(x) &= -(-3) \cdot Bx^{-4} + \left(-\frac{1}{2}\right) \cdot Cx^{-\frac{3}{2}} - 4 \cdot Dx^3 \\ &= \frac{3B}{x^4} - \frac{C}{2\sqrt{x^3}} - 4Dx^3. \end{aligned}$$

6. a. The child's height satisfies $h(a) = 6.46a + 72.34$, so by the power rule:

$$\frac{dh}{da} = h'(a) = 6.46 \text{ cm/yr},$$

which is constant. The growth rate is 6.46 cm/yr at all ages, including ages 2 and 6.

b. Since the growth rate is constant, the predicted height at age 11 is $135 + 6.46 = 141.46$ cm.

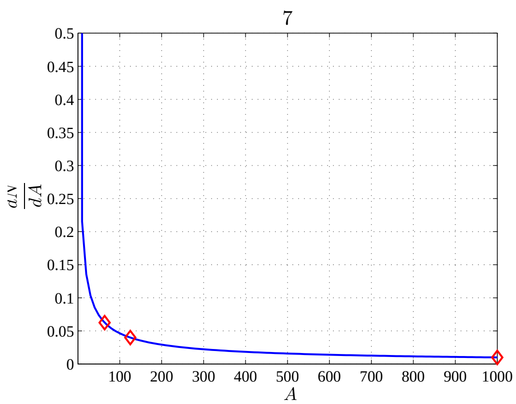
7. Since $N = 3A^{\frac{1}{3}}$, the derivative is

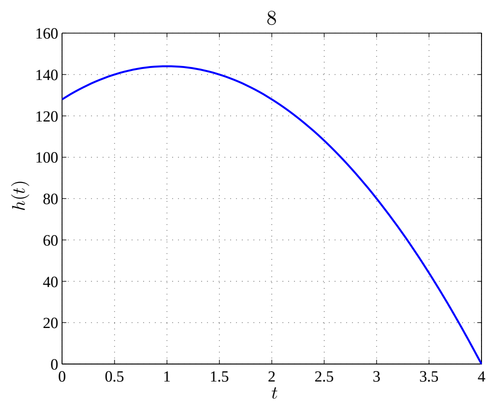
$$N'(A) = 3\left(\frac{1}{3}\right)A^{-\frac{2}{3}} = A^{-\frac{2}{3}}.$$

For the three different areas,

$$\begin{aligned} N'(64) &= 64^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{64^2}} = \frac{1}{4^2} = \frac{1}{16} \\ N'(125) &= 125^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{125^2}} = \frac{1}{5^2} = \frac{1}{25} \\ N'(1000) &= 1000^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{1000^2}} = \frac{1}{10^2} = \frac{1}{100} \end{aligned}$$

The graph of the derivative is shown below to the left.





8. a. The height of the ball satisfies $h(t) = 128 + 32t - 16t^2$, so the velocity is

$$v(t) = h'(t) = 32 - 32t.$$

The maximum height occurs when $v(t) = 0$. But $v(t) = 32 - 32t = 0$ implies $t = 1$. Thus, $h(1) = 128 + 32(1) - 16(1)^2 = 144$ ft is the maximum height of the ball. At $t = 2$ the velocity is $v(2) = 32 - 32(2) = -32$ ft/sec. At $t = 4$ the velocity is $v(4) = 32 - 32(4) = -96$ ft/sec.

b. The graph of the height is shown above to the right. The ball hits the ground when $h(t) = 0$, so

$$h(t) = 128 + 32t - 16t^2 = -16(t^2 - 2t - 8) = -16(t - 4)(t + 2) = 0.$$

It follows that $t = -2$ or $t = 4$. Thus, the ball hits the ground at $t = 4$.

9. a. Since $h(t) = -16t^2 + Vt + 12$, the velocity is $v(t) = h'(t) = -32t + V$ by the power rule.

b. The velocity is zero when $-32t_{max} + V = 0$ or $t_{max} = \frac{V}{32}$.

c. Substituting $t_{max} = \frac{V}{32}$ into the equation for the height of the cat gives

$$h\left(\frac{V}{32}\right) = -16\left(\frac{V}{32}\right)^2 + V\left(\frac{V}{32}\right) + 12 = 16.$$

which gives

$$-\frac{V^2}{64} + \frac{V^2}{32} = 4 \qquad \text{or} \qquad \frac{V^2}{64} = 4.$$

Thus, the initial velocity needs to be $V = 16$ ft/sec, so $v(t) = -32t + 16$. The velocity at $t = 1$ sec is $v(1) = -32(1) + 16 = -16$ ft/sec.

d. With this information, when the cat hits the ground,

$$h(t) = -16t^2 + 16t + 12 = -4(4t^2 - 4t - 3) = -4(2t - 3)(2t + 1) = 0.$$

It follows that either $t = \frac{3}{2}$ or $-\frac{1}{2}$ with the latter not being appropriate. Thus, the cat hits the ground at $t = \frac{3}{2}$ sec with a velocity of $v(1.5) = -32(1.5) + 16 = -32$ ft/sec.

10. a. The population growth is zero at

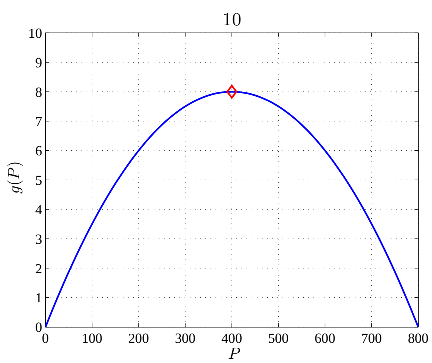
$$g(P_e) = 0.04P_e\left(1 - \frac{P_e}{800}\right) = 0,$$

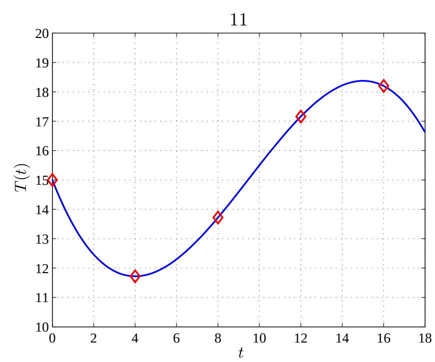
$P_e = 0$ (extinction equilibrium) and $P_e = 800$ (carrying capacity).

b. Begin by writing $g(P)$ in powers of x.

$$\begin{aligned} g(P) &= 0.04P - \frac{0.04P^2}{800}, \\ g'(P) &= 0.04 - \frac{0.04 \cdot 2P}{800} \\ &= 0.04 - 0.0001P. \end{aligned}$$

This has a maximum where $g' = 0$ at $P = 400$ with $g(400) = 8$ individuals/day. This means that the highest growth occurs when there are 400 insects, producing 8 new individuals/day. The graph is shown below on the left.





11. a. Since the body temperature satisfies $T(t) = -0.01t^3 + 0.285t^2 - 1.80t + 15$, the power rule gives the rate of change of body temperature as

$$T'(t) = -0.01 \cdot 3t^2 + 0.285 \cdot 2t - 1.8 = -0.03t^2 + 0.57t - 1.8.$$

b. At midnight, $T'(0) = -0.03(0)^2 + 0.57(0) - 1.8 = -1.8°\text{C/hr}$.

At 4 AM, $T'(4) = -0.03(4)^2 + 0.57(4) - 1.8 = 0°\text{C/hr}$.

At 8 AM, $T'(8) = -0.03(8)^2 + 0.57(8) - 1.8 = 0.84°\text{C/hr}$.

At noon, $T'(12) = -0.03(12)^2 + 0.57(12) - 1.8 = 0.72°\text{C/hr}$.

At 4 PM, $T'(16) = -0.03(16)^2 + 0.57(16) - 1.8 = -0.36°\text{C/hr}$.

The fastest increase in body temperature is at 8 AM, while the most rapid cooling is at midnight (of the given times). The graph is shown above to the right.