1. Consider the discrete logistic growth model given by

$$P_{n+1} = 1.5P_n - 0.0025P_n^2,$$

where P_n is the population after n generations.

a. Suppose that the initial population $P_0 = 50$. Find the population of the next three generations, P_1, P_2, P_3.

b. Find all equilibria for this logistic model.

c. The updating function for this model is

$$F(P) = 1.5P - 0.0025P^2.$$

Find the P-intercepts. Find the coordinates of the vertex, P_v and $F(P_v)$. Sketch a graph of the updating function with the identity map, $P_{n+1} = P_n$.

d. Find the derivative of the updating function, $F'(P)$. Evaluate the derivative at the equilibria and use the value to determine the stability of the equilibrium point(s).

2. Assume that the growth rate of a population P satisfies

$$g(P) = 0.03P\left(1 - \frac{P}{600}\right).$$

The discrete logistic growth model for this population is given by:

$$P_{n+1} = P_n + g(P_n).$$

a. Suppose that the initial population $P_0 = 100$. Find the population of the next three generations, P_1, P_2, P_3.

b. Find the population when the growth rate $g(P)$ is zero (the P-intercepts). The maximum growth occurs at the vertex, which has the coordinates P_v and $g(P_v)$. Find this vertex. You should sketch a graph of the growth function, $g(P)$.

c. Find the equilibria for this logistic model, P_{1e} and P_{2e}. Are the equilibria the same as the values for when the growth rate is zero?

d. The updating function for this model is

$$F(P) = P + g(P).$$

Find the derivative of the updating function, $F'(P)$. Evaluate the derivative at the equilibria and use the value to determine the stability of the equilibrium points, and whether they are monotonic or oscillatory.

3. a. A population of yeast is growing according to the Malthusian growth model

$$P_{n+1} = (1 + r)P_n,$$

where $P_0 = 16$ is the initial population (in 1000/cc) and n is in hours. The population after two hours is found to be $P_2 = 21$ (in 1000/cc). Find the value of r. Determine how long (in hours) it takes for this population to double.

b. After a few hours, the nutrient supply becomes limiting for this culture, so a Logistic growth model better describes the population of yeast. Suppose that experiments show the population follows the model given by

$$P_{n+1} = F(P_n) = 1.1456P_n - 0.0005P_n^2,$$

where again n is in hours and $P_0 = 16$ (in 1000/cc). Find the population of the next three hours, P_1, P_2, P_3. Find all equilibria for this logistic model.

c. The updating function for this model is

$$F(P) = 1.1456P - 0.0005P^2.$$

Find the P-intercepts and the vertex. You should sketch a graph of the updating function with the identity map, $P_{n+1} = P_n$.

d. Find the derivative of the updating function, $F'(P)$. Evaluate the derivative at the equilibria and use the value to determine the stability of the equilibrium point(s) and whether it is monotonic or oscillatory.

4. a. The population of France in 1950 was about 41.8 million, while in 1970, it was about 50.7 million. Assume that the population is growing according to the discrete Malthusian growth equation

$$P_{n+1} = (1 + r)P_n,$$

with $P_0 = 41.8$, where P_0 is the population in 1950 and n is in decades. Use the population in 1970, (P_2), to find the value of r. Determine how many years it takes for this population to double.

b. Estimate the population (in millions) in 2000 based on this model. Given that the population in 2000 was actually 59.4 million, find the percent error between the actual and predicted values.

c. A better model fitting the census data for France is a Logistic growth model given by

$$P_{n+1} = F(P_n) = 1.278P_n - 0.00413P_n^2,$$

where again n is in decades after 1950. If $P_0 = 41.8$, then use this model to predict the populations in 1960 and 1970 (in millions).

d. Find all equilibria for this logistic model for France. Find the derivative of the updating function, $F'(P)$. Evaluate the derivative at the equilibria and use the value to determine the stability of each equilibrium point.

5. Suppose that a population of fish are found to satisfy a Ricker's population model. Let P_n be the population of fish in any year n, with Ricker's model given by

$$P_{n+1} = R(P_n) = aP_ne^{-bP_n}.$$

Suppose that the best fit to a set of data gives $a = 12.2$ and $b = 0.0005$ for the number of fish sampled from a particular river.

a. Let the initial population $P_0 = 300$. Find the population of the next three years, P_1, P_2, P_3.

b. Find all equilibria for this Ricker's model.

c. The updating function for this model is

$$R(P) = 12.2Pe^{-0.0005P}.$$

Find the derivative of the updating function, $R'(P)$. Find the P-intercept. Find the coordinates of the maximum of the updating function, P_{max} and $R(P_{max})$. Find the horizontal asymptote. Sketch a graph of the updating function with the identity map, $P_{n+1} = P_n$.

d. Evaluate the derivative $R'(P_{ne})$ at the equilibria and use the value to determine the stability of each equilibrium point.

6. In fishery management, it is important to know how much fishing can be done without severely harming the population of fish. A modification of Ricker's model that includes fishing is given by the model:

$$P_{n+1} = R(P_n) = 4P_n e^{-0.004P_n} - hP_n,$$

where $a = 4$ and $b = 0.004$ are the constants in Ricker's equation that govern the dynamics of the fish population without any fishing and h is the intensity of harvesting fish (n is in years).

a. Let $h = 0.5$ and $P_0 = 450$. Find the population of fish at this harvesting level for the next three years, P_1, P_2, P_3.

b. With $h = 0.5$, find all equilibria for this Ricker's model. Find the derivative of the updating function, $R'(P)$. Evaluate the derivative at these equilibria and use the value to determine the stability of each equilibrium point.

c. With $h = 1.7$, find the non zero equilibrium for this level of harvesting. Evaluate the derivative at this equilibrium and use the value to determine the stability of the equilibrium point.

d. How intense can the fishing be before this population of fish is driven to extinction? That is, find the value of h that makes the only equilibrium be zero.

7. Hassell's model is often used to study populations of insects. Suppose that the updating function for the population of a species of moth P in a sample plot is given by

$$P_{n+1} = H(P_n) = \frac{25P_n}{(1 + 0.001P_n)^2}.$$

a. Let the initial population $P_0 = 40$. Find the population of the next three years, P_1, P_2, P_3.

b. Find all non-negative equilibria for this Hassell's model.

c. The updating function for this model is

$$H(P) = \frac{25P}{(1 + 0.001P)^2}.$$

Find the derivative of the updating function, $H'(P)$. Find the P-intercept. Find the coordinates of the maximum of the updating function, P_{max} and $H(P_{max})$. Find the horizontal asymptote. Sketch a graph of the updating function with the identity map, $P_{n+1} = P_n$.

d. Evaluate the derivative at the equilibria $H'(P_e)$ and use the value to determine the stability of that equilibrium point

8. The modeling of nerve cells often use a cubic response curve to the membrane potential V. Below we present a overly simple model for the membrane potential at discrete times for a nerve that can be quiescent or have repetitive spiking of action potentials. The simplified model is given by:

$$V_{n+1} = N(V_n) = V_n + 0.06V_n(9 - (V_n - 4)^2).$$

a. Assume that the initial potential is $V_0 = 5$, then determine the membrane potential for the next three time intervals, V_1, V_2, and V_3.

b. Since $N(V_n)$ is a cubic equation, it can have three solutions. Find all equilibria for this model.

c. The updating function is given by

$$N(V) = V + 0.06V\left(9 - (V - 4)^2\right).$$

Find the derivative, $N'(V)$, of the updating function $N(V)$.

d. Evaluate the derivative $N'(V)$ at each of the equilibria found above and determine the local behavior of the solution near each of those equilibria. Think about what your results imply about the behavior of the nerve following different initial stimuli.

9. The San Diego Zoo discovered that because their flamingo population was too small, it would not reproduce until they borrowed some from Sea World. Scientists have discovered that certain gregarious animals require a minimum number of animals in a colony before they reproduce successfully. This is called the *Allee effect*. Consider the following model for the population of a gregarious bird species, where the population, N_n, is given in thousands of birds:

$$N_{n+1} = A(N_n) = N_n + 0.2N_n\left(1 - \frac{1}{16}(N_n - 6)^2\right).$$

a. Assume that the initial population is $N_0 = 4$, then determine the population for the next two generations (N_1 and N_2).

b. Since $A(N_n)$ is a cubic equation, it can have three solutions. Find all equilibria for this model.

c. The updating function is given by

$$A(N) = N + 0.2N\left(1 - \frac{1}{16}(N - 6)^2\right).$$

Find the derivative of the updating function $A(N)$.

d. Evaluate the derivative $A'(N)$ at each of the equilibria found above and determine the local behavior of the solution near each of those equilibria.

e. If we start with an initial population of 2 (thousand), then does this population of birds go to Extinction or Carrying Capacity? If we start with an initial population of 8 (thousand), then does this population of birds go to Extinction or Carrying Capacity? Think what this means ecologically from a conservation perspective for a species that has this Allee effect.