

1. Consider the following linear discrete dynamical model:

$$y_{n+1} = 0.7y_n + 6.$$

Let  $y_0 = 10$ . Find  $y_1$ ,  $y_2$ , and  $y_3$ . Also, find the equilibrium point,  $y_e$ . Does the solution approach the equilibrium (stable) or move away from the equilibrium (unstable)?

2. Consider the following linear discrete dynamical model:

$$z_{n+1} = 1.2z_n - 20.$$

Let  $z_0 = 50$ . Find  $z_1$ ,  $z_2$ , and  $z_3$ . Also, find the equilibrium point,  $z_e$ . Does the solution approach the equilibrium (stable) or move away from the equilibrium (unstable)?

3. Consider the model for breathing with Helium gas (He) as a tracer in the lungs. In the atmosphere, He occurs at 5.2 ppm, so this gives  $\gamma = 5.2$  ppm. Suppose a subject with emphysema begins with a concentration of  $c_0 = 100$  ppm. The mathematical model is the same as in the lecture notes,

$$c_{n+1} = (1 - q)c_n + q\gamma.$$

This subject has  $q = 0.1$ . Find  $c_1$ ,  $c_2$ , and  $c_3$ . Also, find the equilibrium point,  $c_e$ . Does the solution approach the equilibrium (stable) or move away from the equilibrium (unstable)?

4. The lecture notes showed how the model could be used to determine the vital capacity of a subject. Suppose that the tidal volume,  $V_i$ , of the subject is 400 ml. For this experiment, Nitrogen,  $N_2$ , is used to determine the functional reserve capacity,  $V_r$ . (Note that  $V_r = (1 - q)\frac{V_i}{q}$ .) The mathematical model gives

$$c_{n+1} = (1 - q)c_n + q\gamma,$$

where  $\gamma = 0.78$ . You are given that  $c_0 = 0.68$  and  $c_1 = 0.694$ . Use this information to find  $q$ , then determine the functional reserve capacity,  $V_r$ . With this information, determine the amount of  $N_2$  in the lungs in the next two breaths ( $c_2$  and  $c_3$ ). Also, find the equilibrium point,  $c_e$ .

5. Consider a model with immigration given by

$$P_{n+1} = 1.05P_n + 200,$$

with an initial population of  $P_0 = 1000$ . Find the populations at the next three time intervals,  $P_1$ ,  $P_2$ , and  $P_3$ .

6. Below are data on several populations of herbivores in related areas.

$P_0$	$P_1$
70	90
100	150
150	250

The data is assumed to fit a discrete Malthusian model with emigration in the form

$$P_{n+1} = (1 + r)P_n - \mu,$$

where  $r$  is the growth rate and  $\mu$  is the emigration rate.

- Use the data to determine the updating function for this population, *i.e.*, find  $r$  and  $\mu$  and write the equation for this model.
- Beginning with  $P_0 = 100$ , find the populations  $P_1$ ,  $P_2$ , and  $P_3$ .
- Find the equilibrium value  $P_e$  and determine the stability of this equilibrium.

7. Below are data on the population of a species of moth that inhabits an island and breeds annually (then dies).

Year	Moths
1990	6000
1991	5500
1992	5100

If its offspring have a survival rate  $r$ , and there is a net (constant) influx of new moths from surrounding islands entering at a rate  $\mu$ , then the population model has the form

$$P_{n+1} = rP_n + \mu.$$

- From the data below determine the updating function for this population, *i.e.*, find  $r$  and  $\mu$ . Then use this updating function to find the population of moths in 1993, 1994, and 1995.
- Find all equilibria for this model. Based on your iterations in Part a, what is the stability of the equilibria? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.) What does this model predict will ultimately happen to the population of moths?
- On a separate piece of paper, graph the updating function along with the identity map,  $P_{n+1} = P_n$ . Determine all points of intersection  $(P_n, P_{n+1})$ .