1. The initial population is 1,000,000, and the growth constant is $k = 0.03$, so the solution is

$$P(t) = 1,000,000e^{0.03t}.$$

The doubling time is computed by solving the equation $2,000,000 = 1,000,000e^{0.03t_d}$ or $e^{0.03t_d} = 2$. Thus, $t_d = \frac{100}{3}\ln(2) \approx 23.105$ min. The population at $t = 60$ is $P(60) = 1,000,000e^{0.03\cdot 60} = 6,049,647$.

2. The initial population is 500, and the time to double is 7 years. So the rate constant k satisfies, $500e^{k7} = 1000$ or $e^{7k} = 2$. Thus, $k = \frac{\ln(2)}{7} \approx 0.099021$ yr^{-1}. Therefore, $P(20) = 500e^{0.099021\cdot 20} \approx 3623$.

3. The solution of this radioactive decay problem is $R(t) = 5e^{-kt}$ with a rate constant k. With the half-life of 30 days, $R(30) = 2.5 = 5e^{-30k}$ or $e^{30k} = 2$. Thus, $k = \frac{1}{30}\ln(2) \approx 0.023105$ day^{-1}. With this rate constant, $R(10) = 5e^{-0.023105\cdot 10} \approx 3.9685$ mg.

4. a. Consider the solution $y(t) = ce^{-t} + 1$. Differentiate this solution $y'(t) = -ce^{-t} = -y(t) + 1 = 1 - y$.

b. Consider the solution $y(t) = ce^{-t}$. Differentiate this solution $y'(t) = -ce^{-t} = -y(t) = -y$.

c. Consider the solution $y(t) = t - t^2 + c$. Differentiate this solution $y'(t) = 1 - 2t$.

d. Consider the solution $y(t) = ce^{2t}$. Differentiate this solution $y'(t) = 2ce^{2t} = 2y(t) = 2y$.

e. Consider the solution $y(t) = ce^{t^2}$. Differentiate this solution $y'(t) = ce^{t^2}2t = 2ty(t) = 2ty$.

5. The initial condition $h(0) = 4$ is only satisfied by the answers B, C, and D, eliminating A. If we differentiate B ($h(t) = (2 - 3t)^2$), then

$$h'(t) = 2(2 - 3t)(-3) = -6(2 - 3t) = -6\sqrt{(2 - 3t)^2} = -6\sqrt{h(t)},$$

so B satisfies the differential equation. The bucket empties when $h(t) = 0$ at $t = \frac{2}{3}$ hr. The graph is below: