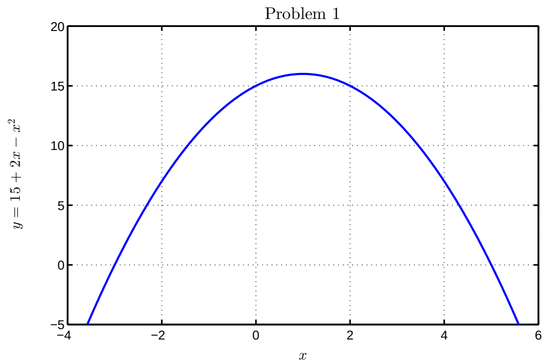
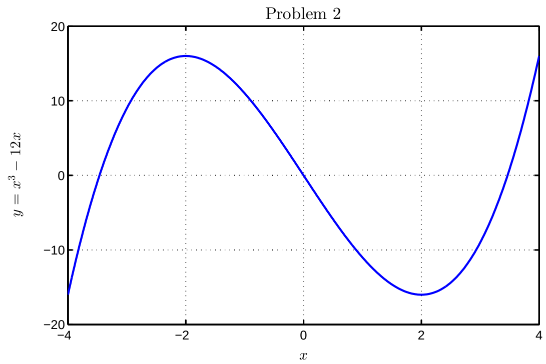
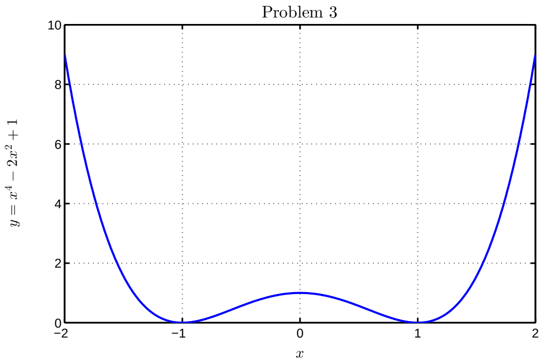
1. Consider $y = 15 + 2x - x^2$, then the first derivative is $y'(x) = 2 - 2x$ and the second derivative is $y''(x) = -2$. The y-intercept satisfies $y(0) = 15$ or $(0, 15)$. The x-intercepts satisfy $15 + 2x - x^2 = (5 - x)(3 + x) = 0$, so $x = -3$ or $x = 5$ The critical point occurs where $y'(x) = 2 - 2x = 0$, which implies $x_c = 1$ and $y(x_c) = 16$. Since $y''(x_c) = -2 < 0$, this is a maximum. The graph appears below to the left.

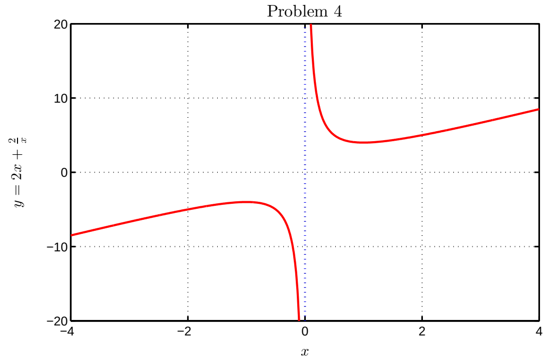




2. Consider $y = x^3 - 12x$, then the first derivative is $y'(x) = 3x^2 - 12$ and the second derivative is $y''(x) = 6x$. The y-intercept satisfies $y(0) = 0$ or $(0, 0)$. The x-intercepts satisfy $y = x^3 - 12x = x(x^2 - 12) = 0$, so $x = 0, \pm 2\sqrt{3}$. Critical points occur where $y'(x) = 3x^2 - 12 = 0$, so $3(x_c - 2)(x_c + 2) = 0$. This implies $x_{1c} = -2$ and $y(x_{1c}) = (-2)^3 - 12(-2) = 16$. Since $y''(x_{1c}) = -12 < 0$, this is a maximum. Also, $x_{2c} = 2$ and $y(x_{2c}) = (2)^3 - 12(2) = -16$. Since $y''(x_{2c}) = 12 > 0$, this is a minimum. There is a point of inflection where $y''(x_p) = 6x = 0$. Therefore $x_p = 0$ and $y(x_p) = 0$. The graph appears above to the right.

3. Consider $y = x^4 - 2x^2 + 1$, then the first derivative is $y'(x) = 4x^3 - 2 \cdot 2x = 4x^3 - 4x$ and the second derivative is $y''(x) = 3 \cdot 4x^2 - 4 = 4(3x^2 - 1)$. The y-intercept satisfies $y(0) = 1$ or $(0, 1)$. The x-intercepts satisfy $y = x^4 - 2x^2 + 1 = (x^2 - 1)(x^2 - 1) = (x + 1)(x - 1)(x + 1)(x - 1) = 0$, so $x = \pm 1$. Critical points occur where $y'(x) = 4x^3 - 4x = 0$, so $4x(x^2 - 1) = 0$ so $x_c = 0, \pm 1$. When $x_{1c} = -1$, then $y(x_{1c}) = (-1)^4 - 2(-1)^2 + 1 = 0$. $y''(x_{1c}) > 0$, so this is a minimum. Also, when $x_{2c} = 0$, then $y(x_{2c}) = (0)^4 - 2(0)^2 + 1 = 1$. $y''(x_{2c}) < 0$, so this is a maximum. When $x_{3c} = 1$, then $y(x_{3c}) = (1)^4 - 2(1)^2 + 1 = 0$. $y''(x_{3c}) > 0$, so this is a minimum. There is a point of inflexion where $y''(x_p) = 4(3x^2 - 1) = 0$. Therefore, $x_p = \pm\frac{1}{\sqrt{3}}$ with $y\left(\pm\frac{1}{\sqrt{3}}\right) = \frac{1}{9} - \frac{2}{3} + 1 = \frac{4}{9}$. The graph appears below to the left.





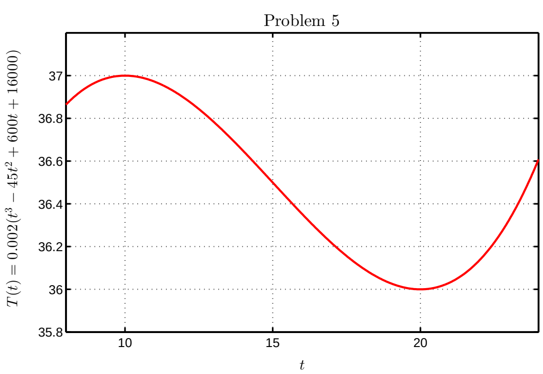
4. Consider $y = 2x + \frac{2}{x}$, then the first derivative is $y'(x) = 2 - 1 \cdot 2x^{-2} = \dfrac{2x^2 - 2}{x^2}$ and the second derivative is $y''(x) = -2 \cdot -2x^{-3} = \dfrac{4}{x^3}$. Since there is a vertical asymptote at $x = 0$, there is no y-intercept. The x-intercepts would satisfy $y = 2x + \dfrac{2}{x} = \dfrac{2x^2 + 2}{x} = 0$, which has no real solution. Thus, there are no x-intercepts. There are no horizontal asymptotes. Critical points occur where $y'(x) = 0$, or $2x^2 - 2 = 2(x+1)(x-1) = 0$, so $x_c = \pm 1$. When $x_{1c} = -1$, then $y(x_{1c}) = 2(-1) + \frac{2}{-1} = -4$. $y''(x_{1c}) < 0$, so this is a maximum. Also, when $x_{2c} = 1$, then $y(x_{2c}) = 2(1) + \frac{2}{1} = 4$. $y''(x_{2c}) > 0$, so this is a minimum. The graph appears above to the right.

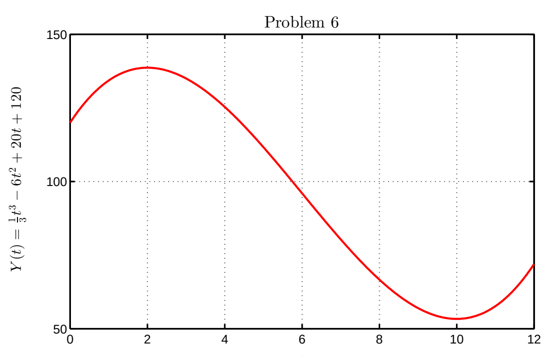
5. a. Since $T(t) = 0.002(t^3 - 45t^2 + 600t + 16000)$, the derivative is $T'(t) = 0.002(3t^2 - 90t + 600) = 0.006t^2 - 0.18t + 1.2$. At noon, $T'(12) = -0.096°\text{C/hr}$.

b. The derivative can be written:

$$\frac{dT}{dt} = 0.006(t^2 - 30t + 200) = 0.006(t - 10)(t - 20).$$

Thus, critical values of t are $t = 10$ and $t = 20$. It is easy to see from the graph that $t = 10$ corresponds to a maximum and $t = 20$ corresponds to a minimum. The maximum temperature of the subject occurs at 10 a.m. with a temperature of 37°C, while the minimum temperature of the subject occurs at 8 p.m. ($t = 20$) with a temperature of 36°C. The graph appears below to the left.





6. a. Since $Y(t) = \frac{1}{3}t^3 - 6t^2 + 20t + 120$, the derivative is given by $Y'(t) = t^2 - 6 \cdot 2t + 20 = t^2 - 12t + 20$. The rate of change in oxygen consumption at $t = 6$ is $Y'(6) = -16$ μl/hr/hr.

b. There are critical points when $Y'(t) = t^2 - 12t + 20 = (t-2)(t-10) = 0$, so $t_c = 2$ and 10. There is a maximum at $t = 2$ with $Y(2) = \frac{416}{3}$ (substituting in the original equation). There is a minimum at $t = 10$ with $Y(10) = \frac{160}{3}$. The graph appears above to the right.

c. The O_2 consumptions at the beginning and end are $Y(0) = 120$ and $Y(12) = 72$. The relative maxima and minima found in (b) are the absolute maxima and minima.

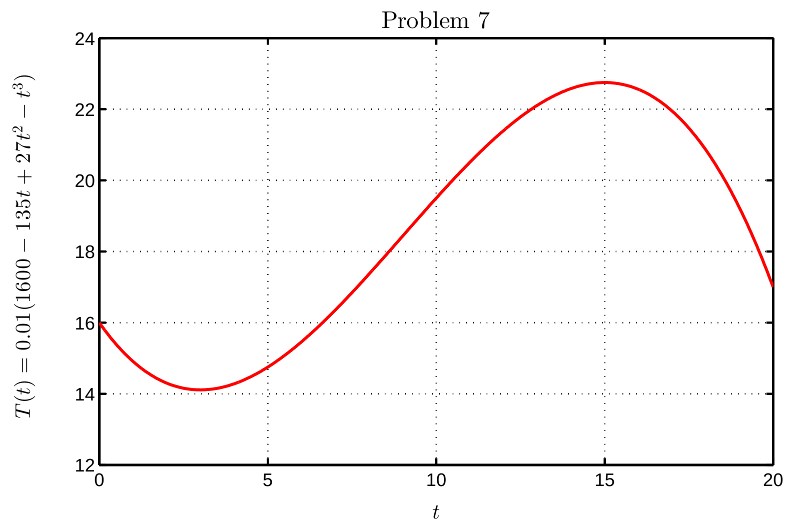
7. a. Consider the function $T(t) = 0.01(1600 - 135t + 27t^2 - t^3)$. The derivative is given by $T'(t) = 0.01(-135 + 54t - 3t^2)$. The rate of change of temperature per hour at $t = 3$ is $T'(3) = 0$.

b. There are critical points when

$$T'(t) = -0.03(t^2 - 18t + 45) = -0.03(t-3)(t-15) = 0,$$

so $t_c = 3$ and 15. There is a minimum at $t = 3$ with $T(3) = 14.11$. There is a maximum at $t = 15$ with $T(15) = 22.75$.

c. The temperatures at the beginning and end of the study are $T(0) = 16$ and $T(20) = 17$. The relative maximum and minimum are the same as the absolute maximum and minimum. The graph appears below.



8. a. The height of the impala satisfies $h(t) = Vt - 490t^2$. Differentiating to obtain the velocity, we have $v(t) = h'(t) = V - 980t$.

b. $v(t) = 0$, when $t_{max} = \frac{V}{980}$. We substitute this value into the original height equation to obtain:

$$h(t_{max}) = V\left(\frac{V}{980}\right) - 490\left(\frac{V}{980}\right)^2 = \frac{V^2}{1960} = 180.$$

It follows that $V^2 = 180(1960)$, so the initial velocity to clear the fence is $V = 420\sqrt{2} \approx 593.97$ cm/sec.

c. By symmetry of the parabola, the time to the t-intercept is twice the time to the maximum. The hang time is $t = 2t_{max} = \frac{V}{490} = \frac{6}{7}\sqrt{2} \approx 1.212$ sec.