Consider the following functions. Find the derivative and second derivative of y. Find the x-intercepts. Find any critical points and decide if they are relative maxima or minima. Find any points of inflection. You should sketch the graph on separate paper.

1. $y = 15 + 2x - x^2$,

2. $y = x^3 - 12x$,

3. $y = x^4 - 2x^2 + 1$,

4. $y = 2x + \frac{2}{x}$.

5. Body temperatures of animals undergo circadian rhythms. A subject's temperature is measured from 8 a.m. until midnight, and his body temperature, T (in °C), is best approximated by the cubic polynomial

$$T(t) = 0.002(t^3 - 45t^2 + 600t + 16000),$$

where t is in hours.

a. Find the rate of change in body temperature $\frac{dT}{dt}$. What is the rate of change in body temperature at noon $(t = 12)$?

b. Use the derivative to find when the maximum temperature of the subject occurs and what that maximum temperature is. Also, use the derivative to find when the minimum temperature of the subject occurs and what that minimum temperature is.

6. In lab we saw the experimental fit of O_2 consumption (in μl/hr) after a blood meal by the beetle *Triatoma phyllosoma*. Below is a cubic polynomial fit to measurements for a different individual "kissing bug,"

$$Y(t) = \frac{1}{3}t^3 - 6t^2 + 20t + 120,$$

where t is in hours, for $0 \le t \le 12$.

a. Find the rate of change in O_2 consumption per hour, $\frac{dY}{dt}$. What is the rate of change in the O_2 consumption at $t = 6$?

b. Use the derivative to find when the relative maximum O_2 consumption for this beetle occurs during the experiment, and what the O_2 consumption is at that time. Use the derivative to find when the relative minimum O_2 consumption for this beetle occurs during the experiment, and what the O_2 consumption is at that time.

c. Find the O_2 consumption at the beginning of the study $(t = 0)$ and at the end $(t = 12)$. Find the time at which the absolute maximum occurs, and the maximum O_2 consumption at that time. Find the time at which the absolute minimum occurs, and the minimum O_2 consumption at that time.

7. Many ecological studies require that the subject studied is correlated with the temperature of the environment (especially insects and plants). Over a 20 hour period, data are collected on the temperature, $T(t)$ in °C. The temperature data are found to best fit the cubic polynomial

$$T(t) = 0.01(1600 - 135t + 27t^2 - t^3),$$

where t is in hours (valid for $0 \le t \le 20$).

a. Find the rate of change in temperature per hour, $\frac{dT}{dt}$. What is the rate of change in the temperature, T', at 3 a.m. $(t = 3)$?

b. Use the derivative to find when the relative minimum temperature occurs during the day, and what the temperature is at that time. Also, use the derivative to find when the relative maximum temperature occurs during the day, and what the temperature is at that time.

c. Find the temperature at the beginning of the study $(t = 0)$, and at the end $(t = 20)$. Find when the absolute maximum temperature occurs during the day, and what the maximum temperature is at that time. Also, find when the absolute minimum temperature occurs during the day, and what the minimum temperature is at that time.

8. a. An impala is migrating across a field that has been fenced with a 180 cm fence. To escape it needs to jump this fence. Assume that the impala jumps the fence with just enough vertical velocity, V to clear it. If the height (in cm) of the impala is given by

$$h(t) = Vt - 490t^2,$$

then find the velocity $v(t) = h'(t)$ of the impala at any time (in sec), $t \geq 0$, before hitting the ground.

b. Find when the velocity is equal to zero in terms of V. This is the time at the maximum height, t_{max}. Since the impala is 180 cm in the air at this time, use the equation for the height, $h(t)$ to compute the initial velocity, V, with which the impala must launch itself to clear the fence.

c. With the initial velocity computed above, determine how long the impala is in the air, when jumping over the fence.