

1. Consider the function $f(x) = 5 - 4 \sin(3x)$. The derivative satisfies

$$f'(x) = -4(3) \cos(3x) = -12 \cos(3x).$$

2. Consider the function $f(x) = 2 \cos(7x) - x^2$. The derivative satisfies

$$f'(x) = -2(7) \sin(7x) - 2x = -14 \sin(7x) - 2x.$$

3. Consider the function $f(x) = 2e^{-6x} + 5 \cos(2(x-9)) - 8 \sin(4(x-4))$. The derivative satisfies

$$f'(x) = -12e^{-6x} - 10 \sin(2(x-9)) - 32 \cos(4(x-4)).$$

4. a. The mass follows $y(t) = 2 \cos(10t)$. Since the cosine function is bounded between -1 and 1 , it follows that the maximum displacements occur with $y(t) = 2$ cm at times when $10t = 2n\pi$ (for any integer $n = 0, 1, 2, \dots$) or $t = \frac{n\pi}{5}$. The minimum displacements occur with $y(t) = -2$ cm at times when $10t = (2n+1)\pi$, where $n = 0, 1, 2, \dots$, or $t = \frac{\pi}{10} + \frac{n\pi}{5}$. The period satisfies $10T = 2\pi$ or $T = \frac{\pi}{5} \approx 0.6283$ sec.

b. The velocity is $v(t) = y'(t) = -20 \sin(10t)$, and the acceleration is $a(t) = v'(t) = -200 \cos(10t)$. The maximum velocity is 20 cm/sec, occurring when $10t = \frac{3\pi}{2} + 2n\pi$, where $n = 0, 1, 2, \dots$, which is equivalent to $t_{max} = \frac{3\pi}{20} + \frac{n\pi}{5} \approx 0.47124$ sec. for $t_{max} \in [0, T)$.

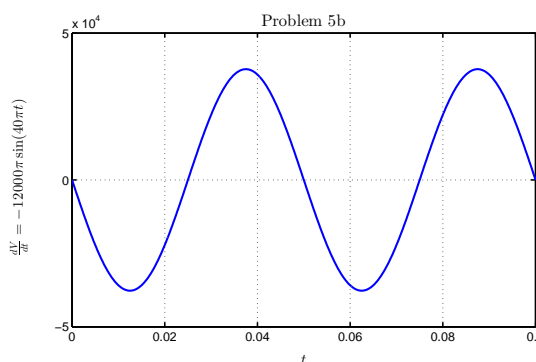
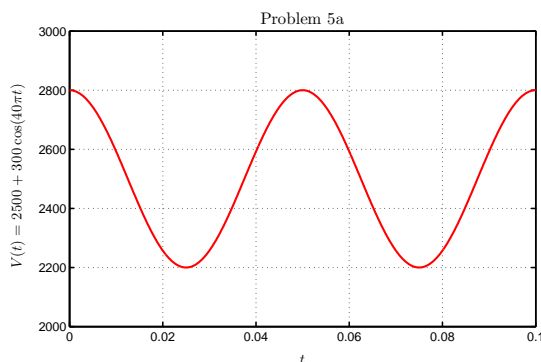
5. a. To create the model for the volume of air in the lungs, we find $A = \frac{(2200+2800)}{2} = 2500$ and $B = 2800 - A = 300$, then solve $\left(\frac{1}{20}\right)\omega = 2\pi$, so $\omega = 40\pi$. Thus, the model becomes

$$V(t) = 2500 + 300 \cos(40\pi t).$$

b. The derivative is $V'(t) = -300(40\pi) \sin(40\pi t) = -12000\pi \sin(40\pi t)$. The maximum rate of exhalation is -12000π ml/min and occurs when $\sin(40\pi t) = 1$ so

$$40\pi t = \frac{\pi}{2} \quad \text{or} \quad t_{max} = \frac{1}{80} = 0.0125 \text{ sec.}$$

The graphs are shown below.



6. a. To create the model for the concentration of FSH, we find $A = \frac{4.3+1.5}{2} = 2.9$ and $B = 4.3 - 2.9 = 1.4$, then solve $28\omega = 2\pi$ or $\omega = \frac{\pi}{14}$. The high concentration occurs at day 9, so $\phi = 9$. Thus, the model is given by:

$$F(t) = 2.9 + 1.4 \cos\left(\frac{\pi(t-9)}{14}\right).$$

On day 14, around the day when ovulation occurs,

$$F(14) = 2.9 + 1.4 \cos\left(\frac{\pi(14-9)}{14}\right) \approx 3.5074.$$

The graph is shown below.

b. From our rules of differentiation $F'(t) = -1.4\left(\frac{\pi}{14}\right) \sin\left(\frac{\pi(t-9)}{14}\right) = -\frac{\pi}{10} \sin\left(\frac{\pi(t-9)}{14}\right)$. We find the rate of change at $t = 14$,

$$F'(14) = -\frac{\pi}{10} \sin\left(\frac{\pi(14-9)}{14}\right) \approx -0.283.$$

