Find the derivatives of the following functions:

1. $f(x) = 5 - 4\sin(3x)$,

2. $f(x) = 2\cos(7x) - x^2$,

3. $f(x) = 2e^{-6x} + 5\cos(2(x-9)) - 8\sin(4(x-4))$.

4. A mass at the end of a spring without any damping executes simple harmonic motion. This motion is described by the equation

$$y(t) = 2\cos(10t),$$

where y is the position of the spring from rest and t is the time.

a. Find the maximum and minimum displacements (positions) of the mass. What is the period of the oscillation, T, for this mass?

b. Find expressions for the velocity ($v(t) = y'(t)$) and the acceleration ($a(t) = y''(t)$) of the mass. Also, determine the maximum velocity of the mass and when it occurs. Choose $t_{max} \in [0, T)$.

5. The lungs do not completely empty or completely fill in normal breathing. The volume of the lungs normally varies between 2200 ml and 2800 ml with a breathing rate of 20 breaths/min. This exchange of air is called the tidal volume.

a. Assume that the volume of air in the lungs satisfies a cosine function written in the following manner:

$$V(t) = A + B\cos(\omega t),$$

where A, B, and ω are constants and t is in seconds. Use the data above and techniques from the previous section to create a model, *i.e.*, find A, B, and ω that simulate the normal breathing of an individual.

b. Differentiate the function above to find the rate of exchange of air in ml/min as a function of time, t. Find the maximum rate of air that is exhaled (loss of volume) and when this occurs during the first 3 seconds. You should sketch a graph of both the volume of air in the lungs and its derivative.

6. During the human female menstrual cycle, the gonadotropin, FSH or follicle stimulating hormone, is released from the pituitary in a sinusoidal manner with a period of approximately 28 days. Guyton's text on Medical Physiology shows that if we define day 0 ($t = 0$) as the beginning of menstruation, then FSH, $F(t)$, cycles with a high concentration of about 4.3 ("relative units") around day 9 and a low concentration of about 1.5 around day 23.

a. Consider a model of the concentration FSH (in "relative units") given by

$$F(t) = A + B\cos(\omega(t - \phi)),$$

where A, B, ω, and ϕ (with $0 \leq \phi < 28$) are constants and t is in days. Use the data above to find the four parameters. If ovulation occurs around day 14, then what is the approximate concentration of FSH at that time ($F(14)$)? You should sketch a graph of the concentration of FSH over one period.

b. Find the derivative of $F(t)$, $F'(t)$. Give its value at the time of ovulation, $F'(14)$. (Don't forget to set your calculator to radians for the evaluations at ovulation.)