1. Consider $f(x) = 2x - 7\ln(x) + e^{2x}$. Using the differentiation rules,

$$f'(x) = 2 - 7\left(\frac{1}{x}\right) + (2)e^{2x} = 2 - \frac{7}{x} + 2e^{2x}.$$

2. Consider $f(x) = 5\ln\left(\frac{1}{x}\right) - e^{-2x} + 2 = 5\ln(x^{-1}) - e^{-2x} + 2 = -5\ln(x) - e^{-2x} + 2$. The rules of differentiation give:

$$f'(x) = -5\left(\frac{1}{x}\right) - (-2)e^{-2x} + 0 = 2e^{-2x} - \frac{5}{x}.$$

3. Consider the function

$$f(x) = \frac{3}{e^{5x}} + 4\ln\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{x} = 3e^{-5x} + 4\ln\left(x^{-\frac{1}{2}}\right) - x^{-1} = 3e^{-5x} - 2\ln(x) - x^{-1}.$$

The rules of differentiation give:

$$f'(x) = 3(-5)e^{-5x} - 2\left(\frac{1}{x}\right) - (-1)x^{-2} = \frac{1}{x^2} - \frac{15}{e^{5x}} - \frac{2}{x}.$$

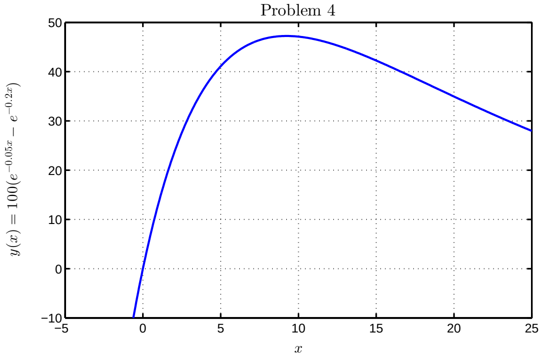
4. Consider the function $y = 100(e^{-0.05x} - e^{-0.2x})$. We take the derivative

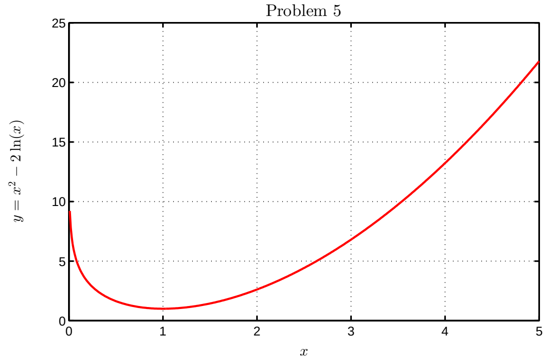
$$y'(x) = 100\left(-0.05e^{-0.05x} - (-0.2)e^{-0.2x}\right) = 5(4e^{-0.2x} - e^{-0.05x}).$$

The domain of $y(x)$ is all x, so there are no vertical asymptotes. The y-intercept is where $x = 0$ or $y = 100(1 - 1) = 0$. This is also the x-intercept, $(0, 0)$. The horizontal asymptote occurs when $x \to +\infty$, when $y \to 0$. The critical point is where $y'(x) = 5(4e^{-0.2x} - e^{-0.05x}) = 0$, so

$$4e^{-0.2x} = e^{-0.05x} \quad \text{or} \quad e^{0.15x} = 4 \quad \text{or} \quad x = \frac{\ln(4)}{0.15} \approx 9.2420.$$

Since $x_c = \frac{\ln(4)}{0.15} =\approx 9.2420$, then $y(x_c) \approx 100(e^{-0.05\cdot 9.242} - e^{-0.2\cdot 9.242}) = 47.25$. This is a relative and absolute maximum. The graph is shown below on the left.





5. Consider the function $y(x) = x^2 - 2\ln(x)$. We take the derivative

$$y'(x) = 2x - 2\left(\frac{1}{x}\right) = 2\left(\frac{x^2-1}{x}\right).$$

Domain for y is $0 < x < +\infty$, so there is clearly no y-intercept. There is a vertical asymptote at $x = 0$ (at the edge of the domain), but no horizontal asymptote. This equation cannot be solved for $y = 0$, so x-intercept must be examined in another way. We will establish that the minimum is positive, so no intercepts of the x-axis exist. Critical points occur when $y'(x) = 0$ or $x^2 - 1 = 0$. Since $x > 0$, then $x_c = 1$. $y(1) = 1$, so there is a minimum at (1,1). Besides being a relative minimum, it is also an absolute minimum. The graph is above to the right.

6. Consider the model $h(t) = 40(e^{-0.005t} - e^{-0.15t})$. The derivative satisfies:

$$h'(t) = 40\left(-0.005e^{-0.005t} - (-0.15)e^{-0.15t}\right) = 6e^{-0.15t} - 0.2e^{-0.005t}.$$

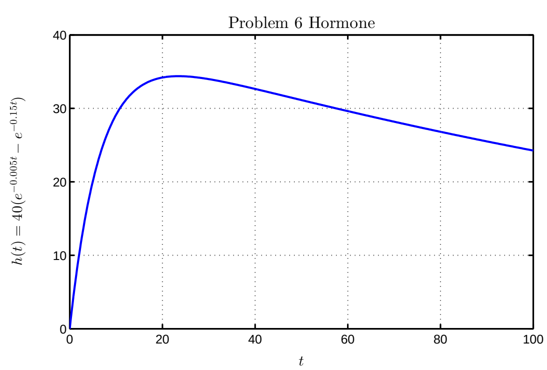
The maximum occurs when $h'(t) = 0$, so

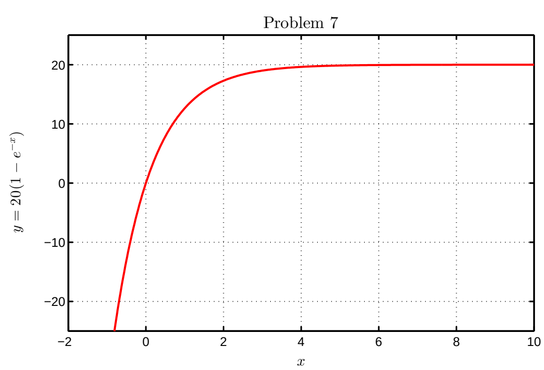
$$6e^{-0.15t} = 0.2e^{-0.005t} \qquad \text{or} \qquad e^{0.145t} = 30 \qquad \text{or} \qquad t_{max} = \frac{\ln(30)}{0.145} \approx 23.46\text{days}.$$

The maximum concentration is $h(t_{max}) = 40\left(e^{-0.005t_{max}} - e^{-0.15t_{max}}\right) \approx 34.39$ ng/dl. We evaluate $h(0) = 40(1-1) = 0$ ng/dl (the h and t-intercept). The horizontal asymptote satisfies:

$$\lim_{t\to+\infty} h(t) = 40(0-0) = 0 \text{ ng/dl}.$$

The graph is shown below to the left.





7. Consider the function $y = 20\left(1 - e^{-x}\right)$. The derivative is

$$y'(x) = 20e^{-x}.$$

The y-intercept is $y(0) = 20(1-1) = 0$, which is also the x-intercept. The horizontal asymptote satisfies

$$\lim_{x\to+\infty} 20\left(1 - e^{-x}\right) = 20(1-0) = 20.$$

The derivative is always positive, so there are no critical points. The graph is shown above on the right.

8. a. The population model is $P(t) = 7.24e^{rt}$, which has the derivative $P'(t) = 7.24re^{rt}$.

b. From the data, $P(10) = 9.64 = 7.24e^{r \cdot 10}$ or $\ln\left(\frac{9.64}{7.24}\right) = 10r$. Thus $r = \frac{\ln\left(\frac{9.64}{7.24}\right)}{10} \approx 0.02863$.

c. Model predicts population in 1860, $P(50) = 7.24e^{0.02863 \cdot 50} = 30.30$ million. Percent error from census data in 1860=$\frac{(30.30-31.4)}{31.4} \cdot 100\% = -3.506\%$.

Model predicts population in 1870, $P(60) = 7.24e^{0.02863 \cdot 60} = 40.343$ million. Percent error from census data in 1870=$\frac{(40.343-39.8)}{39.8} \cdot 100\% = 1.365\%$.

d. Using the expression in Part b. to estimate the annual growth rates in 1860 and 1870,

in 1860, $\frac{dP}{dt} = 7.24 \cdot 0.02863e^{0.02863 \cdot 50} = 0.8675$ million/yr.

In 1870, $\frac{dP}{dt} = 7.24 \cdot 0.02863e^{0.02863 \cdot 60} = 1.155$ million/yr.

The annual growth rate for this decade from the population data is approximately $\frac{39.8-31.4}{10} =$ 0.84 million/yr.

e. The number of years until the population doubled is estimated from

$$7.24 \cdot 2 = 7.24e^{.02863t} \qquad \text{or} \qquad t = \frac{\ln(2)}{0.02863} = 24.21 \text{ years}$$

9. a. The radioactive decay model is $R(t) = 6e^{-kt}$, which has the derivative, $R(t) = -6ke^{-kt}$.

b. The half-life satisfies $R(22) = 3$ μg, so

$$3 = 6e^{-k \cdot 22} \qquad \text{or} \qquad 2 = e^{22k} \qquad \text{or} \qquad k = \frac{\ln(2)}{22} \approx 0.03151.$$

The rate of change at $t = 20$ is $R'(20) = -0.03151 \cdot 6e^{-0.03151 \cdot 20} = -0.1007$ μg/yr, and at $t = 50$ is $R'(50) = -0.03151 \cdot 6e^{-0.03151 \cdot 50} = -0.03912$ μg/yr.

c. For the new sample, $R'(t) = -60e^{-0.03151 \cdot t}$ (losing 60 cpm initially). To find the time until the decay rate is 8 cpm, we solve

$$R'(t) = -60e^{-0.03151 \cdot t} = -8 \qquad \text{or} \qquad e^{0.03151 \cdot t} = \frac{60}{8} \qquad \text{or} \qquad t = \frac{\ln(7.5)}{0.03151} \approx 63.95 \text{ yr.}$$

The age of the 'historic' painting is about 64 years.

10. a. The model satisfies $L(a) = 589 - 375e^{-0.168a}$. The L-intercept occurs when $a = 0$ so $L(0) = 589 - 375 = 214$ mm. The horizontal asymptote occurs as $a \to +\infty$, which yields $L = 589$ mm. The maximum possible length of this fish is thus 589 mm.

b. This fish reaches 90% of its maximum length when $0.9 \cdot 589 = 589 - 375e^{-0.168a}$ or $58.9 = 375e^{-0.168a}$. We solve this for a,

$$e^{0.168a} = \frac{375}{58.9} \qquad \text{or} \qquad a = \frac{\ln\left(\frac{375}{58.9}\right)}{0.168} \approx 11.02 \text{ years.}$$

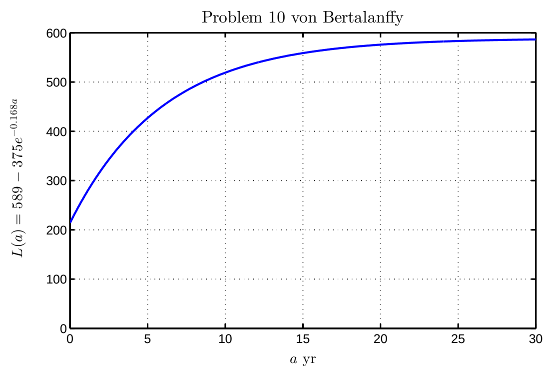
The graph of the von Bertalanffy model is shown below on the left.

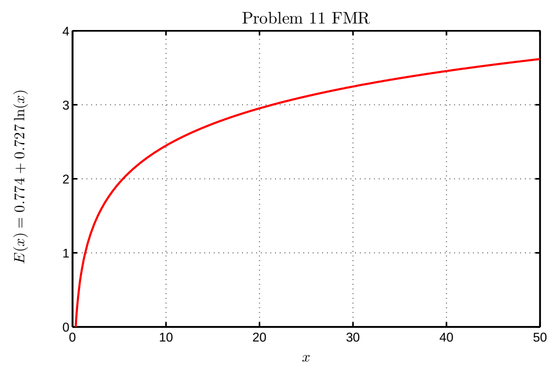
c. Differentiating $L(a)$ with respect to a,

$$L'(a) = -375(-0.168)e^{-0.168a} = 63.0e^{-0.168a}.$$

When the average fish is 5 years old it is growing at a rate of

$$L'(5) = 63.0e^{-0.168 \cdot 5} = 27.2 \text{ mm/year}.$$





11. a. Since $E(x) = 0.774 + 0.727\ln(x)$, it follows that

$$E'(x) = \frac{0.727}{x}.$$

b. The log of the energy expenditure is $E(10,000) = 0.774 + 0.727\ln(10,000) = 7.47$ ln(kJ)/day, while $E'(10,000) = \frac{0.727}{10,000} = 7.27 \times 10^{-5}$ ln(kJ)/day/g.

Biologically, the first result says that a 10 kg pronghorn fawn burns about 7.47 ln(kJ)/day in addition to the energy put into growth. The second result states that each gram of growth adds an additional 7.27×10^{-5} ln(kJ)/day of energy expended when the fawn is near 10 kg in weight. The graph is shown above on the right.

12. a. The soluble iron satisfies $F(t) = 500e^{-0.23t}$. To find when $F(t) = 100$ kg, we solve $100 = 500e^{-0.23t}$, so $\frac{500}{100} = e^{0.23t}$ or $t = \frac{\ln(5)}{0.23} \approx 6.998$ days. $F(t)$ has a horizontal asymptote when $t \to \infty$, which occurs at at $F = 0$. The graph of $F(t)$ is shown above on the right.

b. The derivative is $F'(t) = (-0.23)500e^{-0.23t} = -115e^{-0.23t}$. At $t = 2$, $F(2) = -115e^{-0.23 \cdot 2} = -72.60$ kg/day.

c. The algae population satisfies $P(t) = 10\left(e^{-0.05t} - e^{-0.8t}\right)$. Using the rules of differentiation,

$$P'(t) = 10\left(-0.05e^{-0.05t} - (-0.8)e^{-0.8t}\right) = 8e^{-0.8t} - 0.5e^{-0.05t}.$$

The maximum concentration occurs when $P'(t_{max}) = 0$ so

$$8e^{-0.8t} = 0.5e^{-0.05t} \quad \text{or} \quad e^{0.75t} = 16 \quad \text{or} \quad t_{max} = \frac{\ln(16)}{0.75} \approx 3.697 \text{ days}$$

At that time, the maximum concentration is

$$P(t_{max}) = 10\left(e^{-0.05\cdot3.697} - e^{-0.8\cdot3.697}\right) = 7.793 \text{ (thousands/cc)}.$$

Finally, $P(0) = 10\left(e^0 - e^0\right) = 0$.