Find the derivatives of the following functions.

1. $f(x) = 2x - 7\ln(x) + e^{2x}$,

2. $f(x) = 5\ln\left(\frac{1}{x}\right) - e^{-2x} + 2$,

3. $f(x) = \frac{3}{e^{5x}} + 4\ln\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{x}$.

For each of the functions below, find the derivative. Give the domain of each of the functions. Find the x and y-intercepts and any asymptotes if they exist. Also, list all critical points for each graph and determine if they are maxima or minima. Sketch the curves of the functions.

4. $y = 100(e^{-0.05x} - e^{-0.2x})$,

5. $y = x^2 - 2\ln(x)$.

6. Some hormones have a strong effect on mood, so finding a delivery device that delivers a hormone at a more constant level over a longer period of time is important for hormone therapy. Suppose that a drug company finds a polymer that can be implanted to deliver a hormone, $h(t)$, which is experimentally found to satisfy

$$h(t) = 40\left(e^{-0.005t} - e^{-0.15t}\right),$$

where h is in nanograms per deciliter of blood (ng/dl) and t is in days.

Find the derivative of the function. Find the maximum concentration of this hormone in the body and when this occurs. Evaluate $h(0)$, and find the horizontal asymptote by evaluating $\lim_{t\to\infty} h(t)$. Sketch the graph of the function.

7. Consider the function

$$y = 20\left(1 - e^{-x}\right).$$

Find the derivative. Find the x-intercept. Find the y-intercept. Find the horizontal asymptote, and what x tends to. Find the critical point, if it exists, and decide if this is a relative maximum or minimum. Sketch the curve of the function.

8. a. The population of the United States was about 7.24 million in 1810 and 9.64 million in 1820. If the population $P(t)$ is increasing exponentially, then the population at time t can be described by

$$P(t) = 7.24e^{rt},$$

where P is in millions and t is in years after 1810. Differentiate this function to find the function $\frac{dP}{dt}$ in millions per year.

b. From the data, determine the growth rate r in the expression above.

c. Use the model to predict the population in the year 1860. The actual population was about 31.4 million. What is the percent error between the model and the actual census data? Also, use the model to predict the population in the year 1870. The actual population was about 39.8 million. What is the percent error between the model and the actual census data?

d. Use the expression in Part b. to estimate the annual growth rates in 1860 and 1870 at each of those dates. Take the difference of the populations in 1860 and 1870 and divide by 10 to estimate the annual growth rate for that decade.

e. According to the model, how long until the U. S. population doubled from its 1810 level?

9. White lead, ^{210}Pb, is a radioactive element that appears in the pigment of paints and can be used to date oil paintings. This helps determine modern art forgeries. ^{210}Pb undergoes a $\beta-$ decay to ^{210}Bi. Radioactive substances decay at a rate proportional to the amount of the substance available.

a. Suppose that a 1 g sample of paint contains 6 μg of ^{210}Pb. The amount of ^{210}Pb, $R(t)$ satisfies the equation,

$$R(t) = 6e^{-kt},$$

where the constant k is to be determined. Find $R'(t)$. (Your answer should contain both t and k.)

b. If the half-life of ^{210}Pb is 22 years, then find k. Determine the rate of change in the amount of ^{210}Pb at $t = 20$ and 50 years.

c. Suppose a fresh 1 g sample of pigment gives 60 counts per minute (cpm) (from the $\beta-$ decay of the ^{210}Pb), and a 1 g sample of the same pigment from a historic painting releases 8 cpm, estimate the age of the painting.

10. The cutlassfish is a valuable resource in the marine fishing industry in China. A von Bertalanffy model is fit to data for one species of this fish, giving the length of the fish, $L(t)$ (in mm), as a function of the age, a (in yr). An estimate of the length of this fish is

$$L(a) = 589 - 375e^{-0.168a}.$$

a. Find the L-intercept and any asymptotes. What is the maximum possible length of this fish?

b. Determine how long it takes for this fish to reach 90% of its maximum length. Sketch a graph of the von Bertalanffy model.

c. Differentiate $L(a)$ with respect to a, then determine (in mm/yr) how fast the average fish is growing when it is 5 years old.

11. The log of the field metabolic rate (FMR) or the log of the total energy expenditure per day in excess of growth is calculated for pronghorn fawns using Nagy's formula

$$E(x) = 0.774 + 0.727\ln(x),$$

where x is the mass of the fawn (in g) and $E(x)$ is the log of the energy expenditure (in ln(kJ)/day).

a. Compute the derivative $E'(x)$.

b. Find the log of the energy expenditure when $x = 10,000$, then compute $E'(10,000)$. You should think about the biological interpretation of these results, and sketch the graph of the function.

12. a. It has been shown that iron is the primary limiting nutrient in open ocean waters. There are currently a number of experiments to see if seeding the ocean with iron can create an algal

bloom that fixes CO_2 (to remove this greenhouse gas). Soluble iron that is dumped into the ocean is rapidly used by algae, which are consumed by other organisms. At $t = 0$, a research vessel from Scripps Institute of Oceanography dumps 500 kg of soluble iron. Measurements from a trailing ship indicate that the amount of iron remaining in the water (not in the algae) satisfies the equation:

$$F(t) = 500e^{-0.23t},$$

where t is in days. Find how long it takes for the amount of soluble iron to reach the level of 100 kg remaining. Find when $F(t)$ has a horizontal asymptote. Sketch a graph of $F(t)$.

b. Find the derivative $\frac{dF}{dt}$. Determine the rate of change of soluble iron at $t = 2$.

c. As noted above, the algae rapidly blooms, then fades as the iron passes to organisms higher in the food web. Suppose that samples of the sea water give a population of algae, $P(t)$(in thousands/cc), satisfying the following equation:

$$P(t) = 10\left(e^{-0.05t} - e^{-0.8t}\right),$$

where t is in days. Find the derivative $\frac{dP}{dt}$.Find when the algal population achieves its maximum concentration (t_{max}) and determine what its maximum concentration is. Evaluate $P(0)$.