1. The definite integral satisfies:

$$\begin{aligned}\int_{-1}^{3}(2-x+x^2)\ dx &= \left(2x-\frac{x^2}{2}+\frac{x^3}{3}\right)\Bigg|_{-1}^{3}\\ &= \left(2(3)-\frac{(3)^2}{2}+\frac{(3)^3}{3}\right)-\left(2(-1)-\frac{(-1)^2}{2}+\frac{(-1)^3}{3}\right)\\ &= \frac{40}{3}.\end{aligned}$$

2. The definite integral satisfies:

$$\begin{aligned}\int_{0}^{4}\left(x^2+3-e^{-x}\right)\ dx &= \left(\frac{x^3}{3}+3x+e^{-x}\right)\Bigg|_{0}^{4}\\ &= \left(\frac{4^3}{3}+3(4)+e^{-4}\right)-\left(\frac{0^3}{3}+3(0)+e^{-0}\right)\\ &= \frac{97}{3}+e^{-4}\approx 32.35.\end{aligned}$$

3. We make the substitution $u=2x+6$ with $du=2\ dx$ in definite integral. Note that when $x=-1$, $u=4$, and when $x=5$, $u=16$, so

$$\begin{aligned}\int_{-1}^{5}\frac{dx}{\sqrt{6+2x}} &= \frac{1}{2}\int_{4}^{16}u^{-\frac{1}{2}}du \ = \ \frac{1}{2}\left(2u^{\frac{1}{2}}\right)\Bigg|_{4}^{16}\\ &= \left(\sqrt{16}-\sqrt{4}\right) \ = \ 2.\end{aligned}$$

4. The definite integral satisfies:

$$\begin{aligned}\int_{1}^{5}\frac{x^2+1}{x}dx &= \int_{1}^{5}\left(x+\frac{1}{x}\right)\ dx \ = \ \left(\frac{x^2}{2}+\ln(x)\right)\Bigg|_{1}^{5}\\ &= \left(\frac{5^2}{2}+\ln(5)\right)-\left(\frac{1^2}{2}+\ln(1)\right) = 12+\ln(5)\approx 13.61.\end{aligned}$$

5. The definite integral satisfies:

$$\begin{aligned}\int_{0}^{\pi}(4t+\cos(2t))\ dt &= \left(2t^2+\frac{\sin(2t)}{2}\right)\Bigg|_{0}^{\pi}\\ &= \left(2(\pi)^2+\frac{\sin(2\pi)}{2}\right)-\left(2(0)^2+\frac{\sin(0)}{2}\right) = 2\pi^2.\end{aligned}$$

6. We make the substitution $u = 1 + \sin(x)$ with $du = \cos(x)\ dx$ in definite integral. Note that when $x = 0$, $u = 1$, and when $x = \frac{\pi}{2}$, $u = 2$, so

$$\int_0^{\pi/2} \frac{\cos(x)}{1+\sin(x)} dx = \int_1^2 \left(\frac{1}{u}\right) du = (\ln(u))\Big|_1^2 = (\ln(2)) - (\ln(1)) = \ln(2).$$

7. We make the substitution $u = 25 - x^2$ with $du = -2x\ dx$ in definite integral. Note that when $x = 3$, $u = 16$, and when $x = 4$, $u = 9$, so

$$\begin{aligned} \int_3^4 \frac{2x\ dx}{\sqrt{25-x^2}} &= -\int_{16}^9 \left(u^{-\frac{1}{2}}\right) du = \int_9^{16} \left(u^{-\frac{1}{2}}\right) du \\ &= (2\sqrt{u})\Big|_9^{16} = \left(2\sqrt{16}\right) - \left(2\sqrt{9}\right) = 2. \end{aligned}$$

8. The definite integral satisfies:

$$\begin{aligned} \int_0^{\pi} (9t^2 - \sin(4t))\ dt &= \left(3t^3 + \frac{\cos(4t)}{4}\right)\Big|_0^{\pi} \\ &= \left(3\pi^3 + \frac{\cos(4\pi)}{4}\right) - \left(3(0)^3 + \frac{\cos(4(0))}{4}\right) = 3\pi^3. \end{aligned}$$

9. For the first term we make the substitution $u = x + 3$ with $du = dx$ in definite integral. The first integral has the limits change with $x = -2$ becoming $u = 1$ and $x = 2$ becoming $u = 5$, so

$$\begin{aligned} \int_{-2}^2 \left(\frac{1}{x+3} + e^{2x}\right) dx &= \int_1^5 \left(\frac{1}{u}\right) du + \int_{-2}^2 \left(e^{2x}\right) dx = (\ln(u))\Big|_1^5 + \left(\frac{e^{2x}}{2}\right)\Big|_{-2}^2 \\ &= (\ln(5) - \ln(1)) + \frac{1}{2}\left(e^4 - e^{-4}\right) = \ln(5) + \frac{e^4}{2} - \frac{e^{-4}}{2} \approx 28.899. \end{aligned}$$

10. We make the substitution $u = x^2 + 1$ with $du = 2x\ dx$ in definite integral. The integral has the limits change with $x = 0$ becoming $u = 1$ and $x = 7$ becoming $u = 50$, so

$$\begin{aligned} \int_0^7 \frac{4x}{(x^2+1)^2} dx &= 2\int_1^{50} \left(u^{-2}\right) du = \left(\frac{-2}{u}\right)\Big|_1^{50} \\ &= \left(\frac{-2}{50}\right) - \left(\frac{-2}{1}\right) = \frac{49}{25} = 1.96. \end{aligned}$$

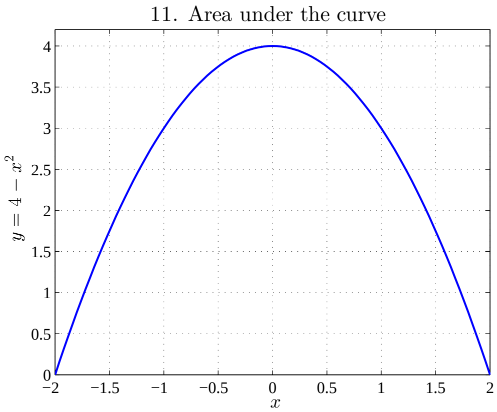
11. The bounded area is given by the definite integral between the x-intercepts. At the x-intercepts,

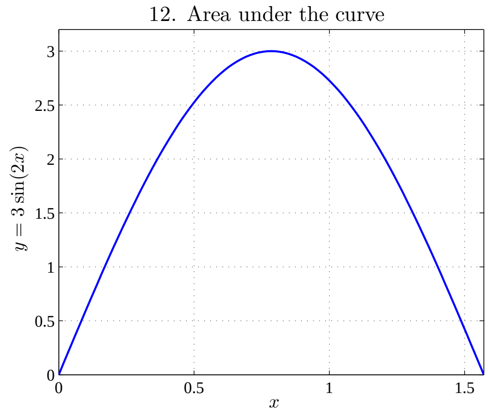
$$y = 0 = 4 - x^2 = (2+x)(2-x) \qquad \text{or} \qquad x = \pm 2.$$

Therefore the area is given by the integral

$$\int_{-2}^{2} \left(4 - x^2\right)\, dx \;=\; \left(4x - \frac{x^3}{3}\right)\Bigg|_{-2}^{2} \;=\; \left(4(2) - \frac{(2)^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right) \;=\; \frac{32}{3}.$$

The graph of the region is shown below on the left.





12. Find the area between the function $y = 3\sin(2x)$ and the x-axis for $0 \le x \le \pi/2$. The bounded area is

$$\int_{0}^{\frac{\pi}{2}} 3\sin(2x)\, dx = \left(-\frac{3}{2}\cos(2x)\right)\Bigg|_{0}^{\frac{\pi}{2}} \;=\; \left(-\frac{3}{2}\cos(\pi)\right) - \left(-\frac{3}{2}\cos(0)\right) \;=3.$$

The graph of the region is shown above on the right.

13. Consider the curves $y = x + 3$ and $y = x^2 + x - 6$.

a. The line has a slope of $m = 1$. The y-intercept is $y = 3$, and the x-intercept is $x = -3$.

For the parabola, the y-intercept is $y = -6$. Solving

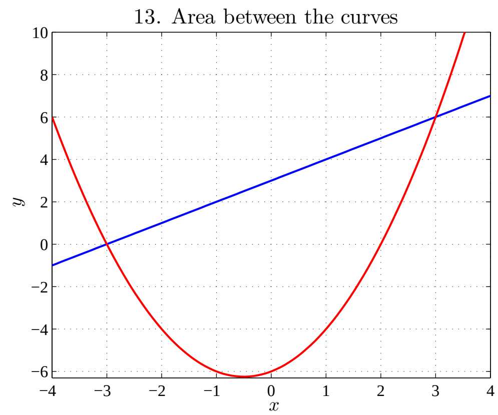
$$x^2 + x - 6 = (x+3)(x-2) = 0 \qquad \text{gives} \qquad x = -3, 2,$$

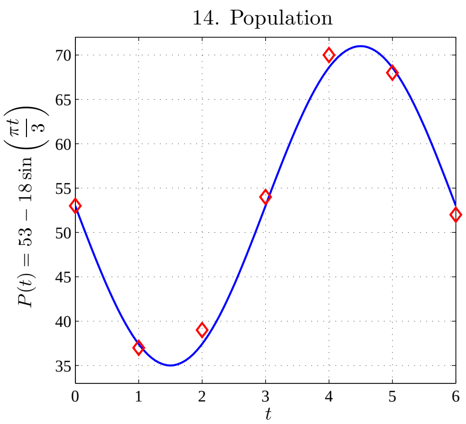
so the x-intercepts are $(-3, 0)$ and $(2, 0)$. The vertex satisfies $x_v = -\frac{1}{2}$ and $y_v = -6.25$. The graph of these curves is shown below on the left.

b. For the intersections, solve $x + 3 = x^2 + x - 6$ or $x^2 - 9 = (x+3)(x-3) = 0$, so $x = \pm 3$. It follows that the points of intersection are $(-3, 0)$ and $(3, 6)$.

c. The area between the curves is the area under the line but over the parabola between the points of intersection. The appropriate integral is

$$\begin{aligned}\int_{-3}^{3} \left((x+3) - (x^2 + x - 6)\right) dx \;&=\; \int_{-3}^{3} \left(9 - x^2\right) dx \;=\; \left(9x - \frac{x^3}{3}\right)\Bigg|_{-3}^{3} \\ &=\; \left(9(3) - \frac{(3)^3}{3}\right) - \left(9(-3) - \frac{(-3)^3}{3}\right) \;=\; 36.\end{aligned}$$





14. a. The average population is

$$\frac{53+37+39+54+70+68+52}{7} = \frac{373}{7} \approx 53.286.$$

b. These data are fitted pretty well by the function

$$P(t) = 53 - 18\sin\left(\frac{\pi}{3}t\right).$$

The maximum population occurs when the sine function is at -1, and the minimum occurs when the sine function is at 1. Thus, the maximum occurs when

$$\frac{\pi t}{3} = \frac{3\pi}{2} \qquad \text{or} \qquad t = \frac{9}{2},$$

so $P(9/2) = 53 + 18 = 71$. Similarly, the minimum occurs when

$$\frac{\pi t}{3} = \frac{\pi}{2} \qquad \text{or} \qquad t = \frac{3}{2},$$

so $P(3/2) = 53 - 18 = 35$. Thus, the maximum population is at $t_{max} = \frac{9}{2}$ and is $P_{max} = 71$. The minimum population is at $t_{min} = \frac{3}{2}$ and is $P_{min} = 35$. The graph of this curve is shown above on the right.

c. Computing the definite integral,

$$P_{ave} = \frac{1}{6}\int_0^6 P(t)dt.$$

$$\begin{aligned} P_{ave} &= \frac{1}{6}\int_0^6 P(t)\,dt = \frac{1}{6}\int_0^6 \left(53 - 18\sin\left(\frac{\pi}{3}t\right)\right)\,dt = \frac{1}{6}\left(53t + \frac{18\cdot 3}{\pi}\cos\left(\frac{\pi}{3}t\right)\right)\Big|_0^6 \\ &= \frac{1}{6}\left(53\cdot 6 + \frac{53}{\pi}\cos(2\pi)\right) - \frac{1}{6}\left(53\cdot 0 + \frac{53}{\pi}\cos(0)\right) = 53. \end{aligned}$$

The average population from this model is 53, which is obvious as this covers one period of this sine function.

15. a. Consider the population model

$$P(t) = \frac{1}{4}t^4 - 3t^3 + 9t^2 + 12.$$

The minimum and maximum populations occur when $P'(t) = 0$, so

$$P'(t) = t^3 - 9t^2 + 18t = t(t^2 - 9t + 18) = t(t-3)(t-6) = 0.$$

It follows that extrema occur at $t = 0$, 3, and 6. The model evaluated at these values gives

$$P(0) = 12 \qquad \text{and} \qquad P(3) = 32.25 \qquad \text{and} \qquad P(6) = 12.$$

Thus, there are two minima at (0, 12) and (6, 12) and one maximum at (3, 32.25).

b. The average population satisfies:

$$\begin{aligned} P_{ave} &= \frac{1}{7}\int_0^7 \left(\frac{1}{4}t^4 - 3t^3 + 9t^2 + 12\right)\, dt = \frac{1}{7}\left(\frac{1}{20}t^5 - \frac{3}{4}t^4 + 3t^3 + 12t\right)\Big|_0^7 \\ &= \frac{1}{7}\left(\frac{7^5}{20} - \frac{3\cdot 7^4}{4} + 3\cdot 7^3 + 12\cdot 7\right) = 21.8. \end{aligned}$$

Thus, the average population using this model is 21.8.

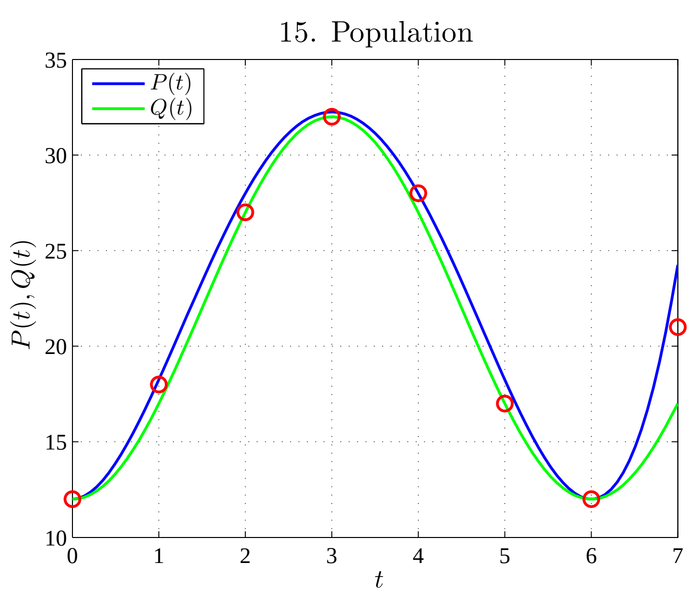
c. The second model is:

$$Q(t) = 22 - 10\cos\left(\frac{\pi}{3}t\right).$$

The minimum and maximum populations occur when the cosine function is 1 and -1, respectively. For $t \in [0, 7]$, the cosine is 1 at $t = 0$ and 2π, while it is -1 when $t = \pi$. Evaluating the population model at these times gives:

$$Q(0) = 12 \qquad \text{and} \qquad Q(\pi) = 32 \qquad \text{and} \qquad P(2\pi) = 12.$$

Thus, there are two minima at $(0, 12)$ and $(2\pi, 12)$ and one maximum at $(\pi, 32)$. A graph of this curve is shown below.



d. The average population with $Q(t)$ is

$$\begin{aligned} Q_{ave} &= \frac{1}{7}\int_0^7 \left(22 - 10\cos\left(\frac{\pi}{3}t\right)\right)\,dt = \frac{1}{7}\left(22t - \frac{30}{\pi}\sin\left(\frac{\pi}{3}t\right)\right)\Big|_0^7 \\ &= \frac{1}{7}\left(\left(22(7) - \frac{30}{\pi}\sin\left(\frac{\pi}{3}7\right)\right) - \left(22(0) - \frac{30}{\pi}\sin\left(\frac{\pi}{3}0\right)\right)\right) = 22 - \frac{30}{7\pi}\left(\frac{\sqrt{3}}{2}\right) \approx 20.82. \end{aligned}$$

The average using this model is 20.82.