1. a. If $e^a = 3.7$ and $e^b = 0.4$, then

$$\begin{aligned}\frac{(e^0+e^a)^2}{e^{a-b}} &= \frac{e^b(1+e^a)^2}{e^a} \\ &= \frac{0.4(1+3.7)^2}{3.7} \\ &= \frac{0.4(4.7)^2}{3.7} = 2.3881.\end{aligned}$$

b. If $e^a = 3.7$ and $e^b = 0.4$, then

$$\begin{aligned}\frac{e^0+e^{2b}}{e^{a+b}} &= \frac{(1+(e^b)^2)}{e^a e^b} \\ &= \frac{(1+(0.4)^2)}{3.7(0.4)} \\ &= \frac{(1.16)}{1.48} = 0.7838.\end{aligned}$$

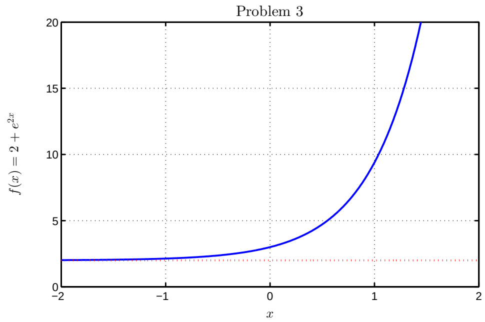
2. a. If $\ln(c) = -1.5$ and $\ln(d) = 2.1$, then

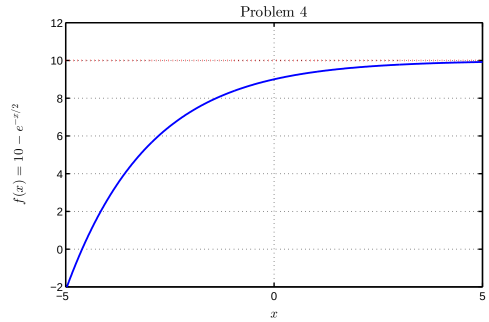
$$\begin{aligned}\frac{\ln(d^2/c)-\ln(e)}{(\ln(cd)+\ln(1))} &= \frac{\ln(d^2)-\ln(c)-1}{\ln(c)+\ln(d)+0} \\ &= \frac{2\ln(d)-\ln(c)-1}{\ln(c)+\ln(d)} \\ &= \frac{2(2.1)-(-1.5)-1}{-1.5+2.1} = \frac{4.7}{0.6} = 7.8333.\end{aligned}$$

b. If $\ln(c) = -1.5$ and $\ln(d) = 2.1$, then

$$\begin{aligned}\frac{\ln(d/c^2)+\ln(1)}{(\ln(e)-\ln(c^3))} &= \frac{\ln(d)-2\ln(c)+0}{1-3\ln(c)} \\ &= \frac{2.1+3}{1+4.5} = \frac{5.1}{5.5} = 0.9273.\end{aligned}$$

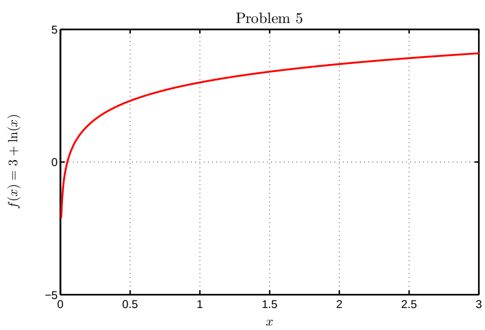
3. The y-intercept satisfies $f(0) = 2 + e^0 = 3$, so it occurs at $(0, 3)$. The x-intercepts solve $f(x) = 0$, but $f(x) = 2 + e^{2x} > 0$, so no x-intercept exists. The exponential is defined for all x, so there are no vertical asymptotes. A horizontal asymptote is found by examining $x \to -\infty$. As $x \to -\infty$, $e^{2x} \to 0$, so $y \to 2 + 0 = 2$. It follows that there is a horizontal asymptote (to the left) at $y = 2$. The graph is shown below to the left.

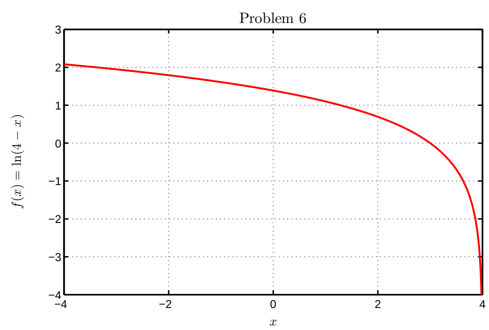




4. The y-intercept satisfies $f(0) = 10 - e^0 = 9$, so it occurs at $(0, 9)$. The x-intercept solves $f(x) = 10 - e^{-x/2} = 0$, so $e^{-x/2} = 10$. By taking the logarithms of both sides, it follows that $-\frac{x}{2} = \ln(10)$, so $x = -2\ln(10) \simeq -4.6052$ and the x-intercept is $(-2\ln(10), 0)$. The exponential is defined for all x, so there are no vertical asymptotes. A horizontal asymptote is found by examining $x \to +\infty$. As $x \to +\infty$, $e^{-x/2} \to 0$, so $y \to 10 + 0 = 10$. It follows that there is a horizontal asymptote (to the right) at $y = 10$. The graph is shown above to the right.

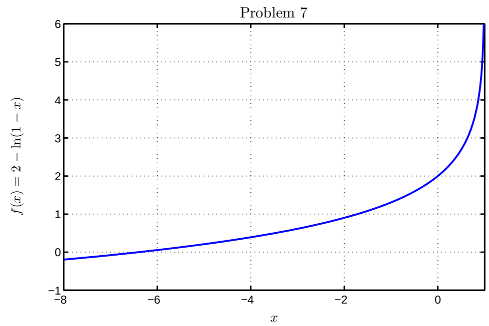
5. The domain requires that the argument of the logarithm is positive, so $x > 0$. The x-intercept satisfies $f(x) = 3 + \ln(x) = 0$, so $\ln(x) = -3$ or $x = e^{-3} = 0.049787$. Thus, the x-intercept occurs at $(e^{-3}, 0)$ The y-intercept is where $x = 0$, but this is outside the domain, hence doesn't exist. A vertical asymptote occurs on the edge of the domain or at $x = 0$. The graph is shown below on the left.

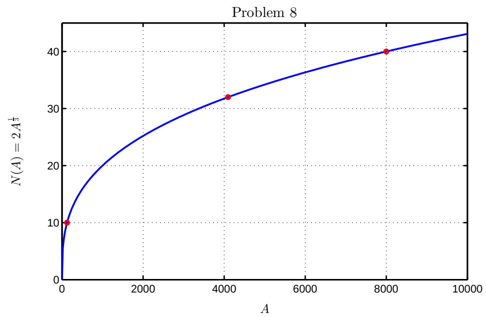




6. The domain requires that the argument of the logarithm is positive, so $4 - x > 0$ or $x < 4$. The x-intercept satisfies $f(x) = \ln(4 - x) = 0$, so $4 - x = 1$. Thus, $x = 3$, and the x-intercept occurs at $(3, 0)$. The y-intercept is where $x = 0$, so $f(0) = \ln(4 - 0) = 1.3863$ or $(0, \ln 4)$. A vertical asymptote occurs on the edge of the domain or at $x = 4$. The graph is shown above on the right.

7. The domain requires that the argument of the logarithm is positive, so $1 - x > 0$ or $x < 1$. The x-intercept satisfies $f(x) = 2 - \ln(1 - x) = 0$, so $\ln(1 - x) = 2$ or $1 - x = e^2$ Thus, $x = 1 - e^2 = -6.3891$, and the x-intercept occurs at $(1 - e^2, 0)$ The y-intercept is where $x = 0$, so $f(0) = 2 - \ln(1 - 0) = 2$ or $(0, 2)$. A vertical asymptote occurs on the edge of the domain or at $x = 1$. The graph is shown below on the left.





8. a. The average number of mammalian species satisfies $N = kA^{\frac{1}{3}}$, so if the islands have areas of 125 and 8000 km^2, then

$$\begin{aligned} N(125) &= 2(125)^{\frac{1}{3}} = 2(5) = 10, \\ N(8000) &= 2(8000)^{\frac{1}{3}} = 2(20) = 40. \end{aligned}$$

b. For 32 species of mammals, the island area satisfies $N = 32 = 2A^{\frac{1}{3}}$, so $16 = A^{\frac{1}{3}}$ and $A = 16^3 = 4096\text{km}^2$. The graph is shown above on the right.

9. From the model, $t = kn^a$, we see that $\ln(t) = \ln(k) + a\ln(n)$, which is a straight line in the logarithms of the data. From the data,

n	$\ln(n)$	t	$\ln(t)$
8	2.0794	360	5.8861
4	1.3863	388.8	5.9631

The slope satisfies

$$a = \frac{\ln(t_2) - \ln(t_1)}{\ln(n_2) - \ln(n_1)} = \frac{5.9631 - 5.8861}{1.3863 - 2.0794} = -0.11103.$$

Similarly, $\ln(k) = \ln(t_1) - a\ln(n_1) = 5.8861 + 0.11103(2.0794) = 6.11699$ or $k = 453.496$. The model can be written:

$$t = 453.496\, n^{-0.11103}.$$

b. Using the values for $n = 2$ and $n = 1$, the winning times predicted by the model are 419.904 seconds (or 6 min 59.9 sec) for pairs and 453.496 sec (or 7 min 33.5 sec) for singles. Complications occur because eights have a coxswain, while fours may or may not have a coxswain. Singles are sculls with two oars for the oarsman, compared to only one oar per oarsman for eights and fours. Pairs are similar to fours, so its time should be the best guess.

10. a. The allometric model satisfies:

$$\begin{aligned} P &= kW^a, \\ \ln(P) &= a\ln(W) + \ln(k). \end{aligned}$$

Note that if $Y = \ln(P)$, $X = \ln(W)$, and $K = \ln(k)$, then this is just the equation of a line

$$Y = aX + K,$$

where X and Y are the logarithms of the data.
The data given is shown with their logarithmic values in the table below:

W	$\ln(W)$	P	$\ln(P)$
4	1.386	615	6.422
28	3.332	350	5.858

From the formula above, the slope is a and satisfies

$$a = \frac{\ln(P_2) - \ln(P_1)}{\ln(W_2) - \ln(W_1)} = \frac{5.858 - 6.422}{3.332 - 1.386} = -0.2897.$$

To obtain k, we see that

$$\ln(k) = \ln(P_1) - a\ln(W_1) = 6.422 + 0.2897(1.386) = 6.823,$$

so

$$k = e^{\ln(k)} = e^{6.823} = 918.9.$$

This gives the allometric model

$$P = 918.9W^{-0.2897}.$$

b. For an 11 gram wren, the allometric model gives:

$$P = 918.9(11)^{-0.2897} = 458.8 \text{ beats/min}.$$

If a dove has a pulse of 130 beats/min, then the allometric model gives

$$\begin{aligned} 130 &= 918.9W^{-0.2897} \\ W^{0.2897} &= \frac{918.9}{130} \\ W &= \left(\frac{918.9}{130}\right)^{1/0.2897} = 855.0 \text{ g} \end{aligned}$$

11. a. The allometric model satisfies:

$$\begin{aligned} T &= kw^a, \\ \ln(T) &= a\ln(w) + \ln(k). \end{aligned}$$

Note that if $Y = \ln(T)$, $X = \ln(w)$, and $K = \ln(k)$, then this is just the equation of a line

$$Y = aX + K,$$

where X and Y are the logarithms of the data.
The data given is shown with their logarithmic values in the table below:

w	$\ln(w)$	T	$\ln(T)$
70	4.248	120	4.787
1.5	0.4055	65	4.174

From the formula above, the slope is a and satisfies

$$a = \frac{\ln(T_2) - \ln(T_1)}{\ln(w_2) - \ln(w_1)} = \frac{4.787 - 4.174}{4.248 - 0.4055} = 0.15954.$$

To obtain k, we see that

$$\ln(k) = \ln(T_1) - a\ln(w_1) = 4.787 - 0.15954(4.248) = 4.1097,$$

so

$$k = e^{\ln(k)} = e^{4.1097} = 60.928.$$

This gives the allometric model

$$T = 60.928w^{0.15954}.$$

b. For a 20 kg dog, the allometric model gives:

$$T = 60.928(20)^{0.15954} = 98.28 \text{ days}.$$

If an animal has erythrocytes with a lifetime of 100 days, then the allometric model gives

$$\begin{aligned} 100 &= 60.928w^{0.15954} \\ w^{0.15954} &= \frac{100}{60.928} \\ w &= \left(\frac{100}{60.928}\right)^{1/0.15954} = 22.32 \text{ kg} \end{aligned}$$