Spring 2010 Math 541

Homework 1

Work the Problems in WeBWorK. In addition on your written part of the HW, answer the question below.

1. The Greek mathematician Archimedes estimated the number π by approximating the circumference of a circle of diameter 1 by the perimeter of both inscribed and circumscribed polygons. The perimeter s_n of the inscribed regular polygon with $2n$ sides and t_n the circumscribed regular polygon with $2n$ sides can be given recursively by the following formulas:

$$s_n < \pi < t_n,$$

where

$$s_{n+1} = \frac{2^{n+1}}{\sqrt{2}}\sqrt{1-\sqrt{1-\left(\frac{s_n}{2^n}\right)^2}}, \qquad s_2 = 2\sqrt{2}$$

and

$$t_{n+1} = \frac{2^{n+1}\left(\sqrt{1+\left(\frac{t_n}{2^n}\right)^2}-1\right)}{\frac{t_n}{2^n}}, \qquad t_2 = 4$$

(a) Calculate s_3 to s_{30}. (Make a good table showing the results.) Describe what you observe in this calculation.

(b) Calculate t_3 to t_{30}. (Make a good table showing the results.) Describe what you observe in this calculation.

(c) What went wrong with the calculation (a) and (b)?

(d) Correct the problem and recompute s_3 to s_{30} and t_3 to t_{30}. With this correction, discuss the convergence to π.