I, ____________________________, pledge that this exam is completely my own work, and that I did not take, borrow or steal any portions from any other person. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.

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Signature

Be sure to show all your work or include a copy of your program.

1. Consider the function given by

$$f(x) = e^{-2x} + 2\,\sin(x) - 1.$$

a. Write a Taylor polynomial of degree 3, $P_3(x)$, and the remainder term $R_3(x)$ about $x_0 = 0$.

b. Use the remainder term to get an upper bound on the error in the approximation, $P_3(x)$, for $x \in [0, 1]$. Find the absolute and relative error between $f(1)$ and $P_3(1)$.

c. One of the roots of this function is $x = 0$. Write Newton's method for finding this root. Show your iterations to a tolerance of 10^{-5}, starting with $x_0 = 0.2$. Is the rate of convergence for this iteration quadratic? Explain.

d. Write down a scheme that converges more rapidly than your Newton's method. Show your iterations for this scheme to a tolerance of 10^{-5}, starting with $x_0 = 0.2$. What is its rate of convergence?

e. Use the **secant method** and **Newton's method** to find the other roots of $f(x) = 0$ for $x \in (0, 8]$. Show your iterations and give the rate of convergence near each of the other roots for each method.

2. Consider the polynomial given by

$$P(x) = x^5 + 3.2\,x^4 - 4.1\,x^3 - 27.5\,x^2 - 36.6\,x - 15.5.$$

a. Use Horner's method to compute $P(2)$ and $P'(2)$.

b. Use Müller's method to find all real and complex roots for $P(x)$.

3. a. The map given by the function:

$$R(x) = 10xe^{-0.05x}$$

has two fixed real fixed points. (This function is often used in population modeling of fish populations.) Can we expect the fixed point iteration (with a "random" starting point for $x \in [0, 100]$) to converge to either of the fixed points? Why or why not?

b. Find the fixed points for this map.

c. The composite map $R \circ R(x) = R(R(x))$ has four fixed points. Find these fixed points. What can you say about the convergence of a "random" starting point near any of these fixed points? If an iteration converges, then give the order of convergence. If it fails to converge, then give your reason for why it fails to converge.

d. For each of the converging iterations in Part c, implement Steffensen's method to accelerate the convergence. Show your program and iterations.

4. Consider the function:

$$f(x) = 2xe^{-0.2x}.$$

Below is a table of the function and its derivative at 4 points

x	$f(x)$	$f'(x)$
0	0	2
2	2.68128	0.804384
4	3.59463	0.179732
6	3.61433	-0.120478

a. Find the Lagrange interpolating polynomial, $P(x)$, through the 4 points given in the table above. Graph both the function and the interpolating polynomial.

b. Use Theorem 3.3 in the text to find a bound on the error for this interpolating polynomial. Compute $f(5)$ and $P(5)$, then calculate the absolute error.

c. A better approximating polynomial is found using Hermite interpolation. Use the techniques from the text to develop a divided-difference table to compute the Hermite polynomial, $H(x)$, then give the Hermite polynomial. Graph both the function and the interpolating polynomial.

d. Use Theorem 3.9 in the text to find a bound on the error for the Hermite interpolating polynomial. Compute $f(5)$ and $H(5)$, then calculate the absolute error.

5. Below are growth data for the Blue Marlin (Makaira mazara)

Age (yr)	1	2	3	5	7	10
Length (cm)	83	155	208	279	319	349

a. Create a Newton's divided-difference table from these data and use it to find the interpolatory polynomial, $P(t)$. Graph the interpolating polynomial and the data.

b. Use this polynomial to estimate the length of a Blue Marlin at ages 4 and 8.

c. Use $P(t)$ to estimate the rate of growth at ages 1, 4, 5, and 8. Take the most accurate three-point differentiation formula to estimate the rate of growth at ages 1 and 5. Compare these values to the ones you found using the derivative of the interpolating polynomial. Which method is more accurate?

d. Find the natural cubic spline fitting the data for the Blue Marlin at ages 1 through 5, and use this spline fit to estimate the length of a Blue Marlin at age 4. Compare this estimate to the one found in Part b.

6. Consider the definite integral given by

$$\int_0^{10} 2xe^{-0.2x}dx.$$

a. Evaluate this integral exactly.

b. Evaluate this integral using the Composite Trapezoid rule with $h = 1$. What is the absolute error for this approximation?

c. Evaluate this integral using the Composite Simpson's rule with $h = 1$. What is the absolute error for this approximation?

d. Determine the values of n and h required to approximate the above integral to a tolerance of 10^{-5} using the Composite Trapezoid rule and compute the approximation.

e. Determine the values of n and h required to approximate the above integral to a tolerance of 10^{-5} using the Composite Simpson's rule and compute the approximation.