1. a. The population of the U. S. in 1980 was about 227 million, and a census in 1990 showed that the population had grown to 249 million. Assume that this population grows according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is the number of decades after 1980, and P_n is population n decades after 1980. Use the data above to find the growth constant r, then write the general solution P_n. Predict the population in the years 2000 and 2020.

b. In 1980, the population of Mexico was 69 million, while in 1990, it had grown to 85 million. Assume its population is also growing according to a Malthusian growth law. Find its rate of growth per decade and predict its population in 2000 and 2020. How long does it take for Mexico's population to double?

c. If these countries continue to grow according to these Malthusian growth laws, then determine the first year when Mexico's population will exceed that of the U. S. and determine their populations at that time.

2. Consider the discrete logistic growth model given by

$$P_{n+1} = f(P_n) = 2.25P_n - 0.00125P_n^2.$$

a. Suppose that the initial population $P_0 = 2000$. Find the population of the next three generations, P_1, P_2, and P_3. Find all equilibria.

b. Sketch a graph of the updating function, $f(P)$, with the identity map, $P_{n+1} = P_n$. Find the intercepts and the vertex of the parabola. Find the equilibria and identify them on your graph.

c. Find the derivative of $f(P)$, then use the results from the lecture notes to determine the behavior near each of the equilibria.

3. A modified version of the discrete logistic growth model that includes emigration is given by

$$P_{n+1} = f(P_n) = 1.1P_n - 0.0001P_n^2 - 9.$$

a. Suppose that the initial population P_0 is 500. Find the population of the next three generations, P_1, P_2, and P_3.

b. Sketch a graph of the updating function with the identity map, $P_{n+1} = P_n$. Be sure to show the intercepts of the parabola as well as the vertex. Find the equilibria and identify them on your graph.

c. Determine the behavior of the solution to this modified logistic growth model near the equilibria.

4. a. The population of France in 1950 was about 41.8 million, while in 1970, it was about 50.8 million. Assume that the population is growing according to the discrete Malthusian growth equation

$$P_{n+1} = (1 + r)P_n, \quad \text{with} \quad P_0 = 41.8,$$

where P_0 is the population in 1950 and n is in decades. Use the population in 1970 (P_2) to find the value of r (to 4 significant figures). Write the formula for the general solution to this model.

b. Estimate the population in 2000 based on this model. Given that the population in 2000 was 59.4million, find the percent error between the actual and predicted values.

c. A better model fitting the census data for France is a Logistic growth model given by

$$P_{n+1} = F(P_n) = 1.28P_n - 0.00416P_n^2,$$

where again n is in decades after 1950. If $P_0 = 41.8$, then use this model to predict the populations in 1960 and 1970.

d. Find the equilibria for this Logistic growth model. Calculate the derivative of $F(P)$ and evaluate it at the larger of the equilibria. What does this value say about the behavior of the solution near this equilibrium?

5. The Chinese mitten crab (*Eriocheir sinensis*) was introduced into the San Francisco Bay either intentionally as an oriental delicacy or by accident in ballast water around 1992. Subsequently, this invasive species has caused widespread economic and environmental damage. Suppose that a 1995 survey of a South Bay estuary found a density of 1.3 crabs/m^2, then a 1997 survey of a South Bay estuary found a density of 2.8 crabs/m^2.

a. Assume that the density of crabs is growing according to the discrete Malthusian growth equation

$$P_{n+1} = (1+r)P_n, \quad \text{with} \quad P_0 = 1.3,$$

where P_0 is the density of crabs in 1995 and n is in years. Use the density of crabs in 1997 to find the value of r (to **4 significant figures**). **Note** that the density of crabs in 1997 is P_2, not P_1. Write the formula for the general solution to this model.

b. Estimate the density of crabs in 2000 based on this model. If the density of crabs in 2000 is 7.1 crabs/m^2, then find the percent error between the actual and predicted values.

c. A Logistic growth model for the density of crabs in this South Bay estuary is given by

$$P_{n+1} = F(P_n) = 1.54P_n - 0.045P_n^2,$$

where again n is in years. If $P_0 = 1.3$ (corresponding again to 1995), then use this model to predict the populations in 1996 and 1997.

d. Find the equilibria for this Logistic growth model. Calculate the derivative of $F(P)$ and evaluate it at the larger of the equilibria. What does this value say about the behavior of the solution near this equilibrium?

6. Let P_n be the population of fish after n months and assume that their population dynamics can be approximated by Ricker's growth model

$$P_{n+1} = R(P_n) = 8P_n e^{-0.004P_n}.$$

a. Assume that the initial population is $P_0 = 100$, then determine the population for the next two generations (P_1 and P_2).

b. From the model above, we see that the updating function is given by

$$R(P) = 8Pe^{-0.004P}.$$

Find $R'(P)$, then determine the maximum of this function (both P and $R(P)$ values). Evaluate the $\lim_{P\to\infty} R(P)$. Sketch its graph for $P \geq 0$.

c. Find all equilibria for Ricker's model and determine the stability of the equilibria. (Give the numerical value of the derivative at the equilibria to justify your stability argument.)

7. In fishery management, it is important to know how much fishing can be done without severely harming the population of fish. A modification of Ricker's model that includes fishing is given by the model:

$$P_{n+1} = R(P_n) = 5P_n e^{-0.001P_n} - hP_n,$$

where h is the intensity of harvesting fish.

a. Let $h = 0.5$ and $P_0 = 100$, then find P_1 and P_2.

b. With $h = 0.5$, find all equilibria for this model. Find the derivative of the updating function and evaluate the derivative at the equilibria. What is the behavior (stability) of these equilibria?

c. How intense can the fishing be before this population of fish is driven to extinction? That is, find the value of h that makes the only equilibrium be zero.

8. Hassell's model is often used for the dynamics of insect populations with discrete generations. Suppose that a population of insects satisfies the discrete model

$$P_{n+1} = H(P_n) = \frac{16P_n}{(1 + 0.005P_n)^2}.$$

a. Suppose $P_0 = 200$ and find the populations in the next two generations, P_1 and P_2.

b. Find the intercepts, extrema, and any asymptotes for $H(P)$ with $P \geq 0$. Sketch a graph of $H(P)$.

c. Find all equilibria for the model above and determine the stability of those equilibria.